Registration No. : | | | | | | | | | | |

Total number of printed pages – 4

B. Tech

BSCM 1205/BSCM 2201(O/N)

Third Semester Examination – 2010

MATHEMATICS – III

(Old and New Course)

Full Marks – 70

Time : 3 Hours

*Answer Question No. 1 which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions precisely : 2×10

   (a) Classify (elliptic, parabolic, hyperbolic) the partial differential equation $u_{xy} + u_x + x = 0$.

   (b) Find the transform $v = \phi(x, y)$ and $z = \psi(x, y)$ which transforms the differential equation $u_{xx} + 2u_{xy} + u_{yy} = 0$.

   (c) Solve the partial differential equation $u_{xy} - u_y = 0$.

   (d) If $z = 1 - i$, then find $z^{\frac{1}{3}}$.

   (e) If $f(z) = \frac{\bar{z}}{z}$, then find $\lim_{z \to 0} f(z)$.

   (f) Find the region described by the relation $\left|\frac{z}{z-1}\right| = 2$.

P.T.O.

(g) If $|z_1| = |z_2|$, then find two complex numbers α and β such that $z_1 = \alpha\beta$ and $z_2 = \alpha\bar{\beta}$.

(h) Evaluate the integration $\int_{|z-1|=2} \frac{dz}{z(z+2)}$.

(i) Find the point at which the function $f(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$ is not conformal.

(j) Find the residue of $f(z) = \frac{1}{(z-1)(z-2)}$ at $z = 3$.

2. Solve the following nonlinear partial differential equations :

(a) Solve $yz - p(xy + q) - qy = 0$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 5

(b) Solve $qs - pt = q^3$ where $s = \frac{\partial^2 z}{\partial x \partial y}$ and $t = \frac{\partial^2 z}{\partial y^2}$. 5

3. Subscripts denote the partial derivative with respect to the subscript variable in the following problems :

(a) Solve $xu_x + u_t = xt$ with $u(x, 0) = u(0, t) = 0$ for $x \geq 0$ and $t \geq 0$ using Laplace transform. 5

(b) Show that $u(x, t) = \int_0^t f(t-\gamma)\,\bar{u}_\gamma\, d\gamma$ will be the temperature distribution in a semi-infinite bar extending from $x = 0$ along the x-axis to ∞ assuming $u(x, 0) = u(\infty, t) = 0$ and $u(0, t) = f(t)$ where $\bar{u}(x, t)$ is the temperature distribution in the same bar when $u(0, t) = 1$. 5

4. Solve the following problem according to the instruction :

(a) Find the steady-state temperature distribution $u(x, y)$ for $0 < x < a$ and $0 < y < b$ in a thin rectangular metal plate in which two faces are insulated, $u(0, y) = u(x, 0) = u(x, b) = 0$ and $u(a, y) = \frac{b}{\pi}$. 6

(b) Find the solution of the partial differential equation $u_{xx} - 4u_{xy} + 3u_{yy} = 0$ by transforming into normal form. 4

5. Solve the following problem according to the instruction :

(a) Find the deflection of a unit circular membrane with initial velocity zero and initial displacement $u(r, 0) = 1 - r^2$. 5

GIET GUNUPUR

(b) A string of length l is stretched and fastened to two fixed points. If the initial displacement is $u(0, x) = \sin\left(\frac{\pi x}{l}\right)$, then find the displacement of the string at any time t. 5

6. Answer the following questions according to the instruction :

(a) Prove that $|z_1 - z_2| \geq |z_1| - |z_2|$. 5

(b) Find the harmonic conjugate of $u(x, y) = \cos(x)\sinh(y)$. 5

7. Answer the following questions according to the instruction :

(a) Find a bilinear transform that maps right half plane to unit disc in complex plane. 5

(b) Find the Laurent series representation of the function $f(z) = \frac{z}{(z-1)(z-3)}$ in the region $0 < |z - 1| < 2$. 5

8. Evaluate the following real integrations using contour :

(a) Evaluate : 5

$$\int_0^{2\pi} e^{\cos(\theta)} \cos\left(\sin(\theta) + n\theta\right) d\theta .$$

(b) Evaluate : 5

$$\int_0^{\infty} \left(\frac{\cos(x)}{\sqrt{x}}\right) dx .$$
