Registration No. : | | | | | | | | | | |

Total number of printed pages – 3

B. Tech
BSCM 1205

Third Semester Examination – 2012-13

MATHEMATICS – III

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) What is the difference between general solution and complete solution of a partial differential equation?

(b) Find the solution of partial differential equation $y^2 u_x - x^2 u_y = 0$, using variable separation.

(c) Express Laplace equation in spherical co-ordinates.

(d) Solve : $\frac{\partial 2z}{\partial x \partial y} = \cos xy$

(e) Check whether the function $f(z) = \ln |z| + i \arg z$ is analytic or not.

(f) Prove that a linear fractional transformation have at most two fixed points.

(g) Define conditional convergence of a series. Give an example.

(h) Find the radius of convergence and region of convergence of the series $\sum_{0}^{\infty} \frac{n^4}{2^n} z^{2n}$.

P.T.O.

(i) Find the Taylor's series expansion of $\frac{1}{z}$ about $z = -1$.

(j) Find the residue of $f(z) = \frac{1}{(z^2+a^2)^2}$ at $z = ai$

2. (a) Find the general solution of $(x^2 - y^2 - z^2)\,p + 2xyq = 2xz$. 5

(b) Using Charpit's method solve the following equation : 5

$$2x^2q^2 + 3yq - 2z + xp = 0$$

3. (a) Solve : $(D^2 + DD' + D' - 1)\,z = e^{x+y} + \sin(x+2y) + 2xy$. 5

(b) Solve $y^2 r - 2ys + t - p - 6y = 0$ using Monge's method. 5

4. (a) Derive the solution of Heat equation for the bar with insulated ends using variable separable method. 4

(b) Find the temperature in a laterally insulated copper bar of length 50 cm long if the initial temperature is $100 \sin(\pi x/80)\,^\circ C$ and the ends are kept at $0^\circ C$. How long will it take for the maximum temperature in the bar to drop to $50^\circ C$. Given, density of copper is $8.92 gm/cm^3$, specific heat 0.092 cal /(gm° C), thermal conuctivity 0.95 cal/(cm sec° C). 6

5. (a) Check the differentiability of a function $f(z) = \frac{z\, re(z)}{|z|}$ at $z = 0$. 5

(b) Prove that if f(z) is an analytic function in a simple connected region, then $\oint f(z)\,dz = 0$, along any closed path C, but the converse is not true by giving suitable example. 5

6. (a) Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)}\,dz$ where $C : |z| = 3$ 5

(b) Evaluate $\oint \frac{e^z}{(z-1)^{2(z^{2+4})}}\,dz$, where C is a circle having center at $z = 0$ and radius 3 unit. 5

7. (a) Find the Laurent series expansion of $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ in the region

(i) $1 < |z-1| < 4$

(ii) $|z-1| > 4$ 5

(b) Check the nature of singularity of the following function : 5

(i) $f(z) = \frac{z - \sin z}{z^2}$,

(ii) $f(z) = e^{\frac{1}{z}}$

(iii) $f(z) = z^3 e^{1/(z-1)}$

8. (a) Evaluate $\oint \frac{30z2 - 23z+5}{(2z-1)^2(3z-1)} dz$; $C : |z| = 1$. 5

(b) Evalute $\int_{-\infty}^{\infty} \frac{x}{(x)^2 - 2x+2)^2} dx$. 5