

Registration No. :

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Total number of printed pages – 3

B. Tech
BSCM 1205

Third Semester Examination – 2013

MATHEMATICS – III

BRANCH : MANUTECH, MME, ENV, CSE, IT, MECH, MARINE, MINERAL,
MINING, AUTO, MANUFACT, EIE, FAT, CHEM, BIOMED, AEIE, IEE,
ICE, BIOTECH, PLASTIC, ELECTRICAL, EEE, ETC, TEXTILE,
FASHION, CIVIL, EC, MM

QUESTION CODE : C-499

Full Marks – 70

Time : 3 Hours

Answer Question No. 1 which is compulsory and any **five** from the rest.

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10
- (a) What is the difference between general solution and complete solution to solve a partial differential equation ?
- (b) Write the steps of Monges method.
- (c) Explain heat equation with initial and boundary conditions.
- (d) What is the solution of a two-dimensional wave equation ?
- (e) Evaluate $\lim_{z \rightarrow 0} \frac{\bar{z}}{|z|}$
- (f) What happens when $ad - bc = 0$ in a linear fractional transformation ?
- (g) Check the convergence of the series $\sum_{n=0}^{\infty} (n+1) z^{n+1}$.
- (h) Find the Taylor series expansion of $f(z) = \frac{1}{z^2}$ about $z = -1$



P.T.O.

- (i) What is the type of singularity of $f(z) = \frac{1 - \sin z}{z^2}$?
- (j) Find the residue of $f(z) = \frac{1 + e^z}{z \cos z + \sin z}$ at its pole.
2. (a) Solve $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$ 5
- (b) Solve $x^2y^2 + p^2q^2 = x^2q^2(x^2 + y^2)$. 5
3. (a) Find the general solution of following differential equation : 5
- $$z_{xx} - z_{xy} - 2z_{yy} = (xy - 1)e^x$$
- 5
- (b) Solve $(x^2D_x^2 - 2xyD_xD_y - 3y^2D_y^2 + xD_x - 3yD_y)z = x^2y \cos(\ln x)$ 5
4. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy -plane, $0 < x < a$ and $0 < y < b$ satisfying the following boundary conditions $u(x, 0) = 0$, $u(x, b) = 0$, $u(0, y) = 0$ and $u(a, y) = f(y)$.
- Solve the above problem in a square of length π and $f(x) = \sin^2 x$, $0 < x < \pi$ 10
5. (a) Check the following functions are analytic or not. 5
- (i) $f(z) = \frac{i}{z^4}$
- (ii) $f(z) = \frac{z \operatorname{Re} z}{|z|}$
- (b) Find the orthogonal trajectories of the following curve $x^4 - 6x^2y^2 + y^4 = c$. 5
6. (a) Evaluate $I = \int_{|z|=\pi} \cos^2 z dz$, along the circle $|z| = \pi$ from $-\pi i$ to πi 5
- (b) Evaluate $I = \int_c \frac{e^{zt}}{(z^2 + 1)^2} dz$, where $c : |z| = 3, t \geq 0$ 5

7. (a) Find the radius of convergence of the following series : 5

(i) $\sum_{n=0}^{\infty} \frac{n+13i}{(2n)} (z-i)^{2n+1}$

(ii) $\frac{(n+2)z^n}{(n+3)(n+5)}$



(b) Find the Laurent series expansion of $f(z) = \frac{7z^2 + 9z - 18}{z^3 - 9z}$ around 5

(i) $z=3$

(ii) in the region $3 < |z-3| < 6$

(iii) $|z-3| \geq 6$

8. (a) Evaluate $I = \int_{-\infty}^{\infty} \frac{x dx}{(x-1)(x^2+2x+2)}$ 5

(b) Evaluate $I = \int_{-\infty}^{\infty} \frac{\sin 4x}{(1+x^2)^2} dx$ 5