Registration no: | | | | | | | | | | |

Total Number of Pages: 2 **B.TECH**
BSCM1205

3rd Semester Regular / Back Examination 2015-16
MATHEMATICS-III
BRANCH: All BTech
Time: 3 Hours
Max marks: 70
Q.CODE: T176

Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.

Q1 Answer the following questions: **(2 x 10)**

a) For $u = e^x \cos y$ find out v such that $f(z) = u + iv$ will be analytic.

b) Determine the image of y = 1 under the mapping $w = z^2$.

c) Find all solutions of $\sin z = 1000$

d) Find the principal value of $(1+i)^{1-i}$.

e) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{n(n+3)}{4^n(n+1)}(z-i)^n$

f) Find the residue of $\frac{z^4}{z^2 - iz + 2}$ at each of its singularities.

g) Solve $u_{xx} = 0$

h) Solve $\sqrt{p} + \sqrt{q} = 1$

i) Solve $u_x - u_y = 0$ by the method of separating the variables.

j) Determine the nodal lines of the solution u_{23} of the two dimension wave equation for a square membrane of side 1.

Q2 **a)** Integrate $|z| + z$ from 1 to i along the unit circle. **(5)**

b) Find the Taylor series of the function $\frac{1}{z}$ at the centre $3i$ and determine its radius of convergence **(5)**

Q3 **a)** i) Integrate the function $\frac{15z+9}{z^3 - 9z}$ counterclockwise around the path $C: |z-3| = 4$ using the residue integration method. **(3+3)**

ii) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+4x^4}$

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{13-5\sin\theta}$ **(4)**

Q4 a) Find the linear fractional transformation that maps 0, i, $-i$ onto $2i$, ∞, ½ + i respectively. **(5)**

b) Solve the partial differential equation $(y-z)p+(x-y)q=(z-x)$ **(5)**

Q5 a) Solve the PDE: $(2D_x+D_y+1)(D_x^2+3D_xD_y-3D_x)z=0$ **(5)**

b) Solve the PDE: $qz-p^2y-q^2y=0$ **(5)**

Q6 a) Find u(x, t) of the string of length L = π when c^2=1, the initial velocity is zero and the initial deflection is 0.1x(x – π). **(5)**

b) Transform the equation $u_{xx}+6u_{xy}+9u_{yy}=0$ to normal form using suitable transform and solve it. **(5)**

Q7 a) Find the temperature u(x,y) in a bar of silver (length 10 cm with constant cross section of area with c = 1) that is perfectly insulated laterally, whose ends are kept at temperature 0^oC and whose initial temperature in (^oC) is f(x) = x(10 – x) **(5)**

b) Find the temperature u in a plate r < 1, y > 0 if the segment –1< x < 1 is kept at 0^oC and the semicircular boundary is kept at constant temperature u_o. **(5)**

Q8 a) Show that the only solution of Laplace equation in spherical polar coordinates depending only on r is u = c/r + d, where c and d are constants. Using this find the electrostatic potential between two concentric spheres of radii r_1 = 2 cm and r_2 = 4 cm kept at the potential 220 volts and 140 volts respectively. **(5)**

b) Solve the given equation by Laplace Transform **(5)**

$x\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}=xt,\ u(x,0)=0,\ u(0,t)=0$