Registration No.: | | | | | | | | | | |

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B.Tech.
PME3D001

3rd Semester Regular Examination 2016-17

APPLIED MATHEMATICS

BRANCH : MECHANICAL

TIME : 3 HOURS
FULL MARKS - 100

QUESTION CODE : Y774

Answer Part - A which is compulsory and any four from Part - B.
The figures in the right-hand margin indicate marks.

Part - A (ANSWER ALL THE QUESTIONS)

1. Answer the following questions precisely. **[2 × 10]**

   (a) Nature of the singular point of $\phi(z) = e^{\frac{1}{z-1}}$ is ....

   (b) Order of the pole $z = 0$ in $\phi(z) = \frac{\sin(z)}{z^4}$ is ....

   (c) Order of the pole $z = 0$ in $\phi(z) = \frac{1-\cos(z)}{z^4}$ is ....

   (d) Residue of $\phi(z) = \cot(z)$ at $z = 0$ is ....

   (e) Residue of $\phi(z) = z\sin\left(\frac{1}{z}\right)$ at $z = \infty$ is ....

   (f) The value of the integral $\int\limits_{|z|=2} \frac{dz}{(z-1)^4}$ is ....

   (g) What is the probability of answering $n-1$ correctly in a two column matching problem containing n information provided all are answered ?

   (h) If $\phi(x) = \frac{1}{2^x}$ for $x - 1, 2, 3, \cdots$ is the probability density function of some random variable X, then what is the moment generating function $G_X(t)$ of the random variable X ?

   (i) If a random variable X having Poisson distribution satisfies $E(X^2) = 6$, what is the value of $E(X)$?

   (j) If X is a random variable with $E(X) = \mu$ and $Var(X) = \sigma^2$, then find $E(Y)$ and $Var(Y)$ where $Y = \frac{X-\mu}{\sigma}$.

2. Answer the following questions in short form. **[2 × 10]**

(a) Find the parameter k so that the the bivariate function $f(x,\, y)$ given below is a joint probability density function of some random variables X and Y.

$$f(x,\, y) = \begin{cases} k, & 0 \le x \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

(b) If a random experiment having two outputs, one being success and other being failure, is conducted until one becomes success, what is the probability density function for being success at xth attempt taking probability of success being p ?

(c) If a random experiment having two outputs, one being success and other being failure, is conducted n times, what is the probability density function for x times success taking probability of success being p ?

(d) Find the nature of the differential equation $u_{xy} + u_x = 0$.

(e) Find the nature of the differential equation $u_{xx} - u_{yy} = 0$.

(f) Find the nature of the differential equation $u_{xx} + u_{yy} = 0$.

(g) Find the residue of $\phi(z) = \frac{7z^2+9z-18}{z(z^2-4)}$ at $z = 3$.

(h) Find the singular points of $\phi(z) = \frac{7z^2+9z-16}{z(z^2-1)}$.

(i) Find the Laurent series of $\phi(z) = \frac{1}{z^2-1}$ about $z = 1$.

(j) Find the Taylor series of $\phi(z) = \frac{1}{z^2-1}$ about $z = 0$.

Part - B (ANSWER ANY FOUR QUESTIONS)

3. Answer in detail in compact form.

(a) A scientist carries all times two match boxes, one in his left pocket and other in his right pocket. The scientist is likely to take a match box from either pockets when he needs. If each box contains n matches, what is the probability that the scientist will find one box empty and other box containing $0 \le r \le n$ matches. **[10]**

(b) Let X be a non-negative integer valued random variable with probability density function $f(x)$ satisfying $(x+1)f(x+1) = (\alpha + \beta x)f(x)$ for $x = 0,\, 1,\, 2,\, \cdots,\, \infty$ and $\beta \neq 1$, then find the expectation $E(X)$. **[5]**

4. Give the detail according to the requirement.

(a) If two points are selected on a line of length L so as to be on opposite sides of the midpoint of the line, then find the probability that the distance between the two points is greater than $\frac{L}{3}$. **[10]**

(b) Let α be the probability that both twins are boys, β be the probability that both twins are girls and when the twins are different sexes the probability that the first born being a girl is $\frac{1}{2}$. If the first born of the twins is a girl, what is the probability the second born is also a girl ? **[5]**

5. Answer the following questions.

(a) If the joint probability density function $f(x, y)$ of the random variables X and Y is given by **[10]**

$$f(x,,y) = \begin{cases} 1, & 0 \le x \le 2,\, 0 \le y \le 1,\, 2y \le x \\ 0, & \text{otherwise} \end{cases},$$

then find the probability density function of the random variable $U = X + Y$.

(b) Determine the parameter a in $f(x)$ given below so that $f(x)$ is the probability density function of some continuous random variable X, and hence find the cumulative distribution function $F(x)$ of X. **[5]**

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ \frac{3}{2}(x-1)^2, & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases},$$

6. Answer the following questions.

(a) Find the solution of the non-linear differential equation $2z + p^2 + qy + 2y^2 = 0$. **[10]**

(b) Find the solution of the linear differential equation $(y^2 + z^2)p - xyq + zx = 0$. **[5]**

7. Answer according to the instruction.

(a) Solve the Laplace equation $u_{xx} + u_{yy} = 0$ subject to the conditions $u(x, b) = x(a - x)$, $u_y(x, 0) = u_x(0, y) = u_x(a, y) = 0$ for $0 \le x \le a$ and $0 \le y \le b$. **[10]**

(b) Find D'Alembert solution for wave equation corresponding to the conditions $u(x, 0) = x$ and $u_t(x, 0) = \sin(x)$ in infinite medium. **[5]**

8. Answer precisely according to requirement.

(a) Evaluate $\int\limits_{|z|=4} \frac{(2z^2+5)\,dz}{(1-z)^3(z^2+3)}$. **[10]**

(b) Evaluate $\int\limits_{|z|=99} \frac{dz}{\prod_{i=1}^{98}(z-i)}$ **[5]**

9. Answer according to the instruction.

(a) Evaluate the real integral $\int_0^{2\pi} \frac{\sin(\theta)\,d\theta}{a+b\sin(\theta)}$ where $a^2 > b^2$. **[10]**

(b) Evaluate the improper integral $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x-1)(x^2+4)}$. **[5]**

END