Registration No: | | | | | | | | | | |

Total Number of Pages: 02

B.TECH
PCS3I001

3rd Semester Regular Examination 2016-17
DISCRETE STRUCTURES
BRANCH: CSE
Time: 3 Hours
Max Marks: 100
Q.CODE: Y579
Answer Part-A which is compulsory and any four from Part-B.
The figures in the right hand margin indicate marks.

Part – A (Answer all the questions)

Q1 **Answer the following questions:** ***dash fill up type*** **(2 x 10)**

a) When the statements p and q both are true then the truth value of the statement $(p \rightarrow q) \wedge \overline{q}$ is _______.

b) $\overline{\exists x\left(P(x) \wedge Q(x)\right)} =$ ________________________

c) On a set S = {a, b, c } with relations defined as R_1 = { (a,b) }, R_2 = { (a, b), (b, c) }, R_3 = { (a, b), (b, a) }, R_4 = { (a, b), (b, c), (c, a) }, the relation that is a transitive is ______.

d) Let R = { (a, b), (b, c), (c, c) } be a relation on the set S = {a, b, c }. The transitive closure of R is ____________________.

e) Let S = { 1, 3, 5, 6, 7, 15, 30} with the partial ordered relation R such that (x, y) ϵ R if x divides y. The maximal elements of S are __________.

f) Let Z be the set of all integers. Let for a, b ϵ Z, a*b = a + b + 1. The identity element for the operation * is _______.

g) Let a group code is given by { 0000000000, 0011111000, 1100000111, 1111111111}. The number of errors it can correct is _______.

h) A complimented distributive lattice is called as __________.

i) Let T be a tree with five vertices. The degree of one of the vertices is 4. The number of leaves in T is _____.

j) In a tree with m edges the number of vertices is ________.

Q2 **Answer the following questions:** ***Short answer type*** **(2 x 10)**

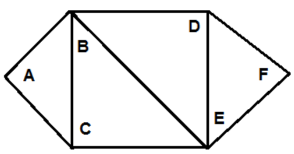
a) Find out the negation of the statement "If 2 + 3 = 6 then 5 + 7 = 57".

b) Write down the truth table for the statement $(p \rightarrow q) \wedge (p \rightarrow r)$.

c) Draw the Hasse diagram for the divisibility relation on the set {1, 3, 7, 11, 21, 33, 77, 231}.

d) Let X = {x, y, z} be a set and let R = { (x,y), (x,z) } be a relation on S. Test whether R is reflexive, symmetric, anti-symmetric or transitive.

e)



Find an Euler path in the given graph.

f) Find the adjacency matrix of the graph given in question number 2(e).

g) Find the chromatic number of the graph given in question number 2(e).

h) Find out the solution of the recurrence relation $a_r = 2a_{r-1},\ a_0 = 5$.

i) Show that in any directed graph the sum of the in-degrees over all the vertices is equal to the sum of the out-degrees over all the vertices.

j) A tree has two vertices of degree 2, one vertex of degree 3 and three vertices of degree 4. Find out the number of vertices of degree one in this tree.

Part – B (Answer any four questions)

Q3 a) Use rules of inferences to provide a formal proof for the following argument **(10)**

$$\bar{a} \to (b \to \bar{c}),\ \bar{a} \vee d,\ \bar{x} \to b,\ \bar{d}\ \therefore c \to x$$

b) Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not equivalent. **(5)**

Q4 a) Prove by mathematical induction that $1 + \frac{1}{4} + \frac{1}{9} + \ldots\ldots + \frac{1}{n^2} < 2 - \frac{1}{n}$ for all positive integer n≥2. **(10)**

b) Let R be a symmetric and transitive relation on a set A. Show that if for every a in A there is b in A such that (a, b) is in R then R is an equivalence relation. **(5)**

Q5 a) Use Warshall's Algorithm to find the transitive closure of R on a set $A = \{1, 2, 3, 4\}$ where $R = \{(2,1), (2,3), (3,1), (3,4), (4,1), (4,3)\}$ **(10)**

b) Solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0,\ a_0 = 2,\ a_1 = 1$ **(5)**

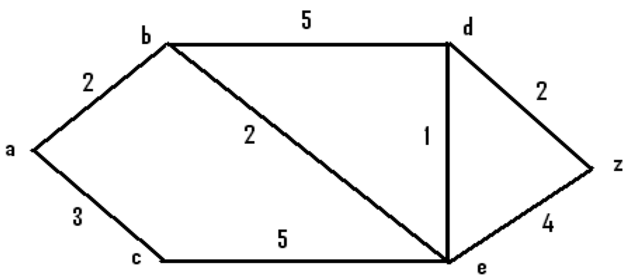
Q6 a) Let $(A, *, \square)$ be an algebraic system with e_1 and e_2 being the identity elements of $*$ and □ respectively. Given that $*$ and □ are distributive over each other show that x ∗ x = x and x□x = x for all x in A. **(10)**

b) Let (A,∗) be a semigroup. For every a,b in A, if a≠b, then a∗b ≠b∗a. Show that for every a in A, a∗a=a. Also show that for a, b in A a∗b∗a = a. **(5)**

Q7 a) Find out the conjunctive normal form and disjunctive normal form of $f(x_1, x_2, x_3) = (x_1 \wedge \bar{x}_2) \vee (\bar{x}_3 \wedge x_2)$ **(10)**

b) Let (A,≤) be a distributive lattice. If $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some a in A, then show that x = y. **(5)**

Q8 a) Find the shortest path and its length between **a** and **z** using Dijkstra's Algorithm. **(10)**



b) Show that in a connected simple planar graph with e edges, v (≥3) vertices and with no circuit of length three then e ≤ 2v – 4. **(5)**

Q9 a) Show that there is a unique path between every two vertices of a tree. Also show that the number of vertices in a tree is one more than the number of edges in the tree. **(10)**

b) Find the minimum spanning tree of the weighted graph given in question 8(a) using Kruskal's Algorithm. **(5)**