Total number of printed pages – 7 B.Tech

BSCM 2102 (Old)/BS 1104 (New)

Second Semester Examination – 2010

MATHEMATICS – II

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) Find Laplace transformation of $f(t) = a^t$.

Or

Find Rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 3 \\ -1 & -4 & 11 \end{pmatrix}$$

P.T.O.

(b) Find inverse Laplace transformation of

$F(s) = \frac{s^2}{s^2+9}$.

Or

Show that every square matrix is sum of symmetric and skew-symmetric matrix

(c) Expand $\sin^2 3x + 3\cos 3x \sin 4x$ in Fourier series over 2π period.

(d) Check which function is even or odd

(i) $f(x) = x \sin x$

(ii) $f(x) = x(1 - \cos x)$.

(e) Prove that $\beta(m, n) = \beta(n, m)$, where β is a beta function.

Or

If $P = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$ is nonsingular matrix and $A = \begin{pmatrix} 5 & 2 \\ -1 & 2 \end{pmatrix}$ any matrix, then find eigen value of $P^{-1}AP$.

(f) Find a vector which is perpendicular to the vectors (–2, 5, 4) and (1, 3 ,2).

(g) Find the projection of $\vec{a} = (3, -2, 1)$ over $\vec{b} = (2, 6, 3)$.

(h) Evaluate the integral $\int_0^\infty e^{-x} x^{\frac{3}{2}}\, dx$.

(i) Find Jacobian $J = \frac{\partial(x, y)}{\partial(u, v)}$,

where $u = x + y$ and $v = xy$.

(j) Evaluate $\int_{(0,0)}^{(1,1)} xy\,ds$, over a curve $y = x$.

2. (a) Solve the differential equations by using Laplace transformation. 5

$y'' + 4y = \sin 2t$, $y(0) = 0$ and $y'(0) = 1$.

Or

Solve the following system of equations :

$2x - 3y + 5z = 15,\ x - 27 + 3z = 9,$

$3x + y + 2z = 6$ by Cramer's Rule.

(b) Find inverse Laplace transformation of the following : 5

(i) $\ln \dfrac{s^2+9}{s^2+1}$

(ii) $\dfrac{se^{-2s}}{s^2+1}$.

Or

Find eigen value and eigen vector of the matrix

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$$

3. (a) Expand the following periodic function in Fourier series $f(x) = 1 + x,\ x \in (0, 2\pi)$. 5

(b) Find sine series of the periodic function $f(x) = x \sin x,\ x \in (0, \pi)$. 5

4. (a) Expand the following periodic function in Fourier series : 5

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 1, & 1 \le x \le 2 \end{cases}$$

(b) Find Fourier Sine transformation of the following : 5

$$f(x) = \begin{cases} \sin 2x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}.$$

Or

Find out what type of conic section the following quadratic form represents and transform it to principal axes :

$$32 = 5x_1^2 - 4x_1x_2 + 3x_2^2$$

5. (a) Find the volume of a Tetrahedron, whose vertices are (1, 2, –2), (3, 3, 5), (2, –1, 3) and (2, 6, –2). 5

(b) Find directional derivative of $f = 3xyz - xy^2$ at (1, 2, 2) in the direction of normal to the surface $x^2 + y^2 - z^2 = 1$, at (1, 1, 1). 5

6. (a) Prove that : 5

$$\text{div}(\mathbf{u} \times \mathbf{v}) = \mathbf{v} \cdot \text{curl}\, \mathbf{u} - \mathbf{u} \cdot \text{curl}\, \mathbf{v}.$$

Where **u** and **v** are vectors.

(b) Evaluate : 5

$$\oint_{(1,2,2)}^{(4,3,4)} yz\,dx + (xz - 2z)\,dy + (xy - 2y + 1)\,dz.$$

7. (a) Evaluate $\int_c x^2y\,dx - xy\,dy$, where c is $y = x$ and $x + y = 2$, from (0, 0) to (2, 0). 5

(b) If **A** is a constant vector and **R** is position vector, prove that curl $(\mathbf{A} \times \mathbf{R}) = 2\mathbf{A}$. 5

8. (a) Evaluate $\iint 5x\,dy\,dz + 3y\,dx\,dz + 7z\,dx\,dy$ over a open box $0 \le x \le 2$, $0 \le y \le 2$, $0 \le z < 2$. 5

(b) Verify Green's theorem $\oint 5y\,dx - xy\,dy$ over a closed curve $y = x$ and $y = x^2$. 5
