

2019

(January)

Time : 3 hours

Full Marks : 80

Answer from **both** the Sections as per direction

*The figures in the right-hand margin indicate marks*

*Candidates are required to answer in their own words  
as far as practicable*

**(MATHEMATICAL METHODS IN PHYSICS)**

SECTION—A

1. Answer any *four* questions of the following : 4 × 4

(a) Find the residue of

$$f(z) = \frac{z \cos z}{(z - \pi)^3} \quad \text{at } z = \pi$$

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(b) Evaluate

$$\int_0^{\infty} \frac{x^{p-1}}{1+x^2} dx, \quad 0 < p < 2$$

by contour integration.

(c) Explain about different types of tensors.

(d) Show that the covariant derivatives of fundamental tensors vanish.

(e) Construct the character table for the group  $D_3$ .

(f) Find the Laplace transform of  $\sin(at)$  and  $\cosh(at)$ .

Or

2. Answer all questions from the following :  $2 \times 8$

(a) Define contour integral.

(b) What are single and multivalued functions?

( 3 )

(c) Define Pseudo tensor.

(d) What is contravariant tensor?

(e) Define group.

(f) What do you mean by irreducible representation?

(g) What is polynomial? Explain.

(h) Define Fourier transform.

### SECTION-B

Answer all questions :  $16 \times 4$

3. (a) Obtain Cauchy-Riemann conditions of a function of a complex variable and evaluate

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz,$$

where  $C$  is a circle  $|z| = 3$ .

( 4 )

Or

- (b) State and prove Cauchy's Residue theorem and evaluate the integral

$$\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)^2}, \quad a > b > 0.$$

4. (a) Explain invariant tensor and epsilon tensors and show that  $\delta'_i{}^j$  is invariant and transforms as mixed tensor of rank two.

Or

- (b) Explain about the Fundamental tensor and show that all the fundamental tensors  $g_{\mu\nu}$ ,  $g^{\mu\nu}$  and  $g_\mu{}^\nu$  are associated tensors.

5. (a) Explain subgroup and show that the order of a subgroup of a finite group is a divisor of the order of the group.

Or

- (b) Explain about the classes in group theory and find the classes of  $D'_3$  group.

( 5 )

6. (a) Obtain the series solution of Bessel's differential equation.

Or

- (b) Obtain Laplace transform of a derivative and show that

$$L\{t \cos t\} = \frac{s^2 - 1}{(s^2 + 1)^2}.$$