Total Pages—5 **MA/M.Sc.—Math-IS(102)**

2019

(January)

Time : 3 hours

Full Marks : 80

Answer from **both** the Sections as per direction

The figures in the right-hand margin indicate marks

Candidates are required to answer in their own words as far as practicable

(TOPOLOGY)

SECTION – A

1. Answer any *four* from the following : 4×4

   (*a*) Prove that if F is a closed set, then CF is an open set.

   (*b*) Show that the union of two topologies for a set need not be a topology for the set.

   (*c*) If every two points of a set are contained in some connected subset of E, then prove that E is a connected set.

(Turn Over)

(*d*) What are the properties of a T_0-space and a T_2-space ?

(*e*) What is a hereditary property of a first axiom space ?

(*f*) What is the difference between box topology and product topology ?

Or

2. Answer *all* the questions : 2×8

(*a*) State Urysohn's metrization theorem.

(*b*) State axioms of constability.

(*c*) What is a base for a topology ?

(*d*) Define lower-limit topology.

(*e*) Define first and second axiom spaces.

(*f*) Define regular and normal spaces.

(*g*) What is regularity ?

(*h*) When a space is completely regular ?

SECTION – B

Answer all questions : 16×4

3. (*a*) (*i*) For any set E in a topological space, prove that $C(E) = E \cup d(E)$.

(*ii*) If E is a subset of a topological space $(X, \mathscr{T})$ and if $d(F) \subseteq E \subseteq F$ for some subset $F \subseteq X$, show that E is a closed set.

Or

(*b*) (*i*) Let X be any uncountable set, and let $\mathscr{T}$ be the family consisting of ϕ and all complements of countable sets. Show that $\mathscr{T}$ is a topology for X.

(*ii*) Prove that Kuratowski closure axioms.

4. (*a*) (*i*) Show that any separation of a topological space must be into two non-empty, disjoint sets which are both open and closed.

(*ii*) Prove that every closed subset of a compact space is compact.

Or

(*b*) (*i*) Prove that a compact subset of a topological space is countably compact.

(*ii*) Show that the space of all ordinals less than the first uncountable ordinal, given the order topology, is countably compact but not compact.

5. (*a*) (*i*) Show that in a T_1-space, no finite set has a limit point.

(*ii*) Prove that a T_1-space is countably compact iff every infinite open covering has a proper subcover.

Or

(*b*) (*i*) If Y is compact, then prove that π_X is a closed mapping of $X \times Y$ onto X.

(*ii*) Show that $\mathscr{D}$ is compact and K is nowhere dense.

6. (*a*) Prove Urysohn's Metrization theorem.

Or

(*b*) (*i*) Prove that $X \times Y$ is compact iff X and Y are compact.

(*ii*) Show that $X \times Y$ has local compactness property iff both X and Y have the same property.