 MA/M.Sc.—Math-IIIS(CE-313)

2019

(January)

Time : 3 hours

Full Marks : 80

Answer from **both** the Sections as per direction

The figures in the right-hand margin indicate marks

Candidates are required to answer in their own words as far as practicable

(ORDINARY DIFFERENTIAL EQUATION - I)

SECTION – A

1. Answer any *four* of the following : 4 × 4

   (*a*) Verify whether the following equation is exact, solve it :

   $$3t^2x^2dt + 3t^2xdx = 0$$

   (*b*) Solve the IVP, for $\frac{\pi}{2} \le t < \pi$

   $$x' + (\cot t)x = 2\operatorname{cosec} t,\ x\left(\frac{\pi}{2}\right) = 1.$$

(*c*) Solve that IVP

$$x'' + x' + 2x = 0, \; x(0) = 0, x'(0) = 2$$

(*d*) Solve

$$x_1' = 2x_1 + x_2$$
$$x_2' = 3x_1 + 4x_2$$

(*e*) Calculate the successive approximations for the IVP $x' = g(t), \; x(0) = 0$.

(*f*) Verify that $y(t) = e^{-t}$ is a solution of $y'(t) + \frac{1}{e} y(t-1) = 0$.

Or

2. Answer *all* questions : 2×8

(*a*) Compute the first two successive approximations for the solution of the following equation $x' = tx, x(0) = 1$.

(*b*) Does the following solar system $x' = ax + f(t)$ admit periodic solution where

(*i*) $a = -1, \; f(t) = \sin\frac{3t}{2}$

(*c*) Find the general solution of $x^{(4)} - 16x = 0$.

(*d*) What is the order and degree of the differential equations ?

(*i*) $5\frac{d^2x}{dt^2} + 2\left(1 - \left(\frac{dx}{dt}\right)^3\right)^{1/2} - x = 0$

(*ii*) $\left(\frac{d^2x}{dt^2}\right)^3 + 7\left(\frac{d^2x}{dt^2}\right)^2 \Big/ \left(\frac{d^2x}{dt^2} + \frac{d^3x}{dt^3}\right) = x$

(*e*) Solve

$$x' = -\frac{\sin t}{\cos x}$$

(*f*) Solve

$$x'' = 3x^{2/3}$$

(*g*) Solve

$$x' + \frac{3x + 2t}{x + 2} = 0$$

(*h*) Solve

$$x' + 2x = 0, \; x(0) = 3.$$

SECTION – B

Answer **all** questions of the following : 16×4

3. (*a*) (*i*) Prove that

$$\sin x, \sin\left(x+\frac{\pi}{8}\right), \sin\left(x-\frac{\pi}{8}\right)$$

are linearly dependent functions on $(-\infty, \infty)$.

(*ii*) Find the solution of the equation $x'' - x = 1$ which vanishes when $t = 0$ and tends to a finite limit as $t \to \infty$.

Or

(*b*) (*i*) Solve

$$x'' + 4x = 8t^2 - 4t + 1$$

(*ii*) Solve

$$x'' + x' = 4t^2 e^t$$

4. (*a*) (*i*) Consider

$$(1-t^2)x'' - 2tx' + 2x = 0,\ 0 < t < 1$$

Given $\phi_1(t) = t$ is a solution to it. Find the second linearly independent solution.

(*ii*) Find the particular solution by using the method of undetermined coefficients.

$$x' - 7x' = (3 - 36t)e^{4t}$$

Or

(*b*) (*i*) Solve the Euler equation by assuming a solution of the form $x(t) = t^r$.

$$6t^2x'' + tx' + x = 0$$

(*ii*) Solve

$$x'' - 9x' + 20x = 0,\ -\infty < t < \infty$$

5. (*a*) (*i*) Find the determinant of fundamental matrix $\phi(t)$ which satisfies $\phi(0) = E$ for the system $x' = Ax$, where

$$A = \begin{bmatrix} -1 & 3 & 4 \\ 0 & 2 & 0 \\ 1 & 5 & 1 \end{bmatrix}$$

(*ii*) Determine $\exp(tA)$ for the system $x' = Ax$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix}$$

Or

(*b*) (*i*) Solve the following systems of equations

$$x_1' = 2x_1 + x_2$$
$$x_2' = 2x_2 + 4x_3$$
$$x_3' = x_3 - x_1$$

(*ii*) Prove that

$$\exp\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} t\right) = \begin{bmatrix} \cos ht & \sin ht \\ \sin ht & \cos ht \end{bmatrix}$$

6. (*a*) Consider the IVP $x'(t) = \frac{1}{1+x^2}$, $x(0) = 0$, $t \geq 0, |x| < \infty$.

(*i*) Show that the IVP has a unique non-local solution on $(0, \infty)$.

(*ii*) Solve the above equation by the method of separation of variables and then show that the solution $x(t)$, with $x(0) = 0$, satisfies $\frac{1}{3}x^3(t) + x(t) - t = 0, t \geq 0$.

Or

(*b*) (*i*) Show that the solution of the following equation is bounded

$$x'(t) = -3x(t) + x(t - r)$$

(*ii*) Show that the solution of the following equation is asymptotically stable

$$x'(t) = -5x(t) + 4x(t - r), \quad (r > 0)$$