Or

(*b*) (*i*) Solve the following matrix game by linear programming :

$$\begin{pmatrix} 1 & 0 & 4 & 1 \\ 0 & -4 & -3 & 1 \end{pmatrix}$$

(*ii*) Use the dominance relation to slove the following matrix game :

$$\begin{pmatrix} 4 & 2 & 2 & 1 \\ 2 & 3 & 5 & 4 \\ 3 & 2 & 1 & 1 \\ 2 & 1 & 2 & 1 \end{pmatrix}$$

2019

(January)

Time : 3 hours

Full Marks : 80

Answer from **both** the Sections as per direction

The figures in the right-hand margin indicate marks

Candidates are required to answer in their own words as far as practicable

(OPTIMIZATION TECHNIQUES)

SECTION—A

1. Answer any *four* of the following : 4 × 4

(*a*) State GOMORY'S mixed integer-continuous variables algorithm.

(*b*) What are Kuhn-Tucker optimality conditions ?

(*c*) Let f be a pseudoconvex function on an open convex set $T \subset \mathbb{R}^n$. Also, let $\nabla f(x^*) = 0$ for some $x^* \in T$. Then what about x^* ?

(*d*) Derive the dual of the primal linear program :

Maximize $v = 3x_1 + 2x_2$

subject to $x_1 + 3x_2 \le 6,$
$x_1 - x_2 \le 3,$
$x_1 + 2x_2 \le 5,$
$x_1 \ge 0,\ x_2 \ge 0.$

(*e*) State Farkas' lemma.

SECTION—B

Answer **all** the questions : 16 × 4

2. (*a*) Use Gomory's algorithm to solve the program :

Minimize $Z = -9x_1 - 10x_2$

subject to $x_1 + x_3 = 3$
$2x_1 + 5x_2 + x_4 = 15,$
$x_1, x_2, x_3, x_4 \ge 0$ and integers.

Or

(*b*) Solve the following problem by the upper bounding technique :

Maximize $v = 3x_1 + 2x_2$

subject to
$x_1 - 3x_2 \le 3,\quad 2x_1 + x_2 \le 20,\ -x_1 + x_2 \le 6,$
$x_1 - 2x_2 \le 4,\quad x_1 + 3x_2 \le 30,$
$0 \le x_1 \le 8,\ 0 \le x_2 \le 6.$

3. (*a*) (*i*) Let f be a pseudo convex function on an open convex set $T \subset \mathbb{R}^n$. Also, let $\nabla f(x^*) = 0$ for some $x^* \in T$. Then prove that x^* is a global minimum of f over T.

(*ii*) Let f is differentiable at $x^* \in T$ and x^* is a local minimum of the problem Minimize $f(x)$, subject to $x \in T$, then prove that

$$\nabla f(x^*)^T d \ge 0 \text{ for all } d \in \overline{D}(x^*).$$

Or

(*b*) (*i*) Solve the following problem using Lagrangian multipliers :
Minimize $f(x) = 3x_1^2 + 4x_2^2 + 5x_3^2$
subject to $x_1 + x_2 + x_3 = 10.$

(*ii*) Find a direction $z \in \mathbb{R}^n$ along which the directional derivative of the function f at a point $x \in \mathbb{R}^n$ is a minimum.

4. (*a*) Consider the convex programming problem :

Minimize $(-x_1 + x_2)$

subject to $x_1^2 + x_2 \leq 0$

$x_2 \geq 0.$

Show that $x_1 = 0, x_2 = 0$ is the optimal solution to this program. Also, is Slater's constraint qualification satisfied at $(0, 0)^T$? Further, show that the Lagrangian of the problem has no saddle point.

Or

(*b*) (*i*) Prove that the Lagrangian associated with the problem :

Minimize (x^2)

subject to $0 \leq x \leq 1$

must have a Saddle point and then find it.

(*ii*) Apply the *K-T* conditions to solve the following problem :

Minimize $(x_1^2 + x_2)$

subject to $x_1^2 + x_2^2 - 9 = 0$

$-(x_1 + x_2^2) + 1 \geq 0$

$x_1 + x_2 \leq 1$

5. (*a*) (*i*) Use the dominance relations to solve the following matrix game :

$$\begin{pmatrix} 2 & 0 & 3 \\ 3 & -1 & 1 \\ 5 & 2 & -1 \end{pmatrix}$$

(*ii*) Solve the game with the following pay-off matrix by the graphical method.

$$\begin{pmatrix} 2 & -3 & -1 & 1 \\ 0 & 2 & 1 & 2 \end{pmatrix}$$