Or

(*b*) (*i*) Solve the difference equation

$$\Delta^2 y_n + 3\Delta y_n - 4 y_n = n^2$$

with the initial conditions $y_0 = 0, y_2 = 2$.

(*ii*) Use the classical Runge-Kutta formula of fourth order to find the numerical solution at $x = 0{\cdot}8$ for

$$\frac{dy}{dx} = \sqrt{x+y},\ y(0\cdot4) = 0\cdot41$$

Assume the step length $h = 0{\cdot}2$.

MA/M.Sc.—Math-IS (105)

2019

(January)

Time : 3 hours

Full Marks : 80

Answer from **both** the Sections as directed

The figures in the right-hand margin indicate marks

Candidates are required to answer in their own words as far as practicable

(NUMERICAL ANALYSIS AND ITS APPLICATIONS)

SECTION – A

. Answer any *four* of the following : 4 × 4

(*a*) Find the unique polynomial of degree 2 or less, such that $f(0) = 1, f(1) = 3, f(3) = 55$, using Lagrange interpolation.

(*b*) Prove that $\nabla - \Delta = -\Delta\nabla$.

(*c*) Find the Jacobian Matrix for the system of equations :

$$f_1(x, y) = x^2 + y^2 - x = 0$$
$$f_2(x, y) = x^2 - y^2 - y = 0,$$

using the methods

$$\left(\frac{\partial f}{\partial x}\right)_{(x_i, y_j)} = \frac{f_{i+1,j} - f_{i-1,j}}{2h}$$

$$\left(\frac{\partial f}{\partial y}\right)_{(x_i, y_j)} = \frac{f_{i,j+1} - f_{i,j-1}}{2k}$$

with $h = k = 1$.

(*d*) Evaluate the integral

$$I = \int_0^1 \frac{dx}{1+x}$$

using trapezoidal rule.

(*e*) Find the approximate value of

$$I = \int_0^1 \frac{dx}{1+x}$$

using Simpson's rule.

(*f*) Obtain polynomial approximation $p(x)$ to $f(x) = e^{-x}$ using the Taylor's expansion about $x_0 = 0$.

Or

2. Answer any *eight* questions : 2 × 8

(*a*) What do you mean by interpolation ?

(*b*) State Hermite polynomial.

(*c*) Why we need different approximation methods ?

(*d*) What is stirling interpolation ?

(*e*) What is least square approximation ?

(*f*) State Chebyshev polynomial.

(*g*) What is uniform approximation ?

(*h*) What is Gram-Schmidt orthogonalizing process ?

(*i*) What is piece wise interpolation ?

(*j*) What are the integration methods based on undetermined coefficients ?

SECTION – B

Answer all questions : 16 × 4

3. (*a*) (*i*) Let $f(x) = \log(1 + x)$, $x_0 = 1$ and $x_1 = 1 \cdot 1$. Calculate an approximate value for $f(1 \cdot 04)$ using linear interpolation and obtain a bound on the truncation error.

(*ii*) Calculate the *n*th divided difference of $f(x) = \frac{1}{x}$.

Or

(*b*) (*i*) If $f(x) = e^{ax}$, show that

$$\Delta^n f(x) = (e^{ah} - 1)^n e^{ax}.$$

(*ii*) Find the unique polynomial $p(x)$ of degree 2 or less such that $p(1) = 1$, $p(3) = 27$, $p(4) = 64$ using Aitken's iterated interpolation formula. Evaluate $p(1 \cdot 5)$.

4. (*a*) (*i*) Determine the least-squares approximation of the type $ax^2 + bx + c$, to the function 2^x at the points $x_i = 0, 1, 2, 3, 4$.

(*ii*) The Bernstein polynomial of degree *n* approximating a function $f(x)$ defined in (0, 1) is given by

$$B_n(f, x) = \sum_{m=0}^{n} f\left(\frac{m}{n}\right) \binom{n}{m} x^m (1-x)^{n-m}$$

Prove that

$$\frac{d}{dx} B_n\left(x^3, x\right) = 3x^2 + \frac{3x(2-3x)}{n} + \frac{(1-6x+6x^2)}{n^2}, n \geq 3.$$

Or

(*b*) Using the following data obtain the Lagrange and Newton's bivariate interpolating polynomials :

$y \backslash x$	0	1	2
0	1	3	7
1	3	6	11
2	7	11	17

5. (*a*) Define :

$$S(h) = \frac{-y(x+2h)+4y(x+h)-3y(x)}{2h}.$$

(*i*) Show that $y'(x) - S(h) = C_1h^2 + C_2h^3 + C_3h^4 + \ldots$ and state C_1.

(*ii*) Calculate $y'(0{\cdot}398)$ as accurately as possible using the table below and with the aid of the approximation $S(h)$. Give an error estimate (the values in the table are correctly rounded).

x	0·398	0·399	0·400	0·401	0·402
$y(x)$	0·408591	0·409671	0·410752	0·411834	0·412915

Or

(*b*) Compute $I_p = \int_0^1 \frac{x^p}{x^3+12}\,dx$ for $p = 0, 1$ using trapezoidal and Simpson's rules with the number of points 3, 5 and 9. Improve the results using Romberg integration.

6. (*a*) (*i*) Find the coefficients a and b in the operator formula

$$\delta^2 + a\delta^4 = h^2D^2(1 + b\delta^2) + o(h^8).$$

(*ii*) Show that this formula defines an explicit multistep method for the integration of the special second order differential equation $y'' = f(x, y)$.