Registration No. : | | | | | | | | | | |

Total number of printed pages – 4

B.Tech
BS 1104

Second Semester Examination – 2012

MATHEMATICS – II

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest. The figures in the right-hand margin indicate marks.*

1. Answer the following questions : 2×10

(a) Give example of a function that has no Laplace Transform. Indicate the reason.

(b) If $L(y(t)) = Y(s)$, find $L\left\{\int_0^t y(\tau)\cos(t-\tau)\,d\tau\right\}$.

(c) Temperature at a point (x, y, z) in space is given by $f(x, y, z) = xyz$. If particle P at $(-1, 1, 3)$ is to be moved to $(0, -1, 5)$ on a straight line path, find the rate of change of temperature experienced by P.

(d) When a vector field is said to be conservative ? Show whether the vector function $\vec{v}(x,y,z)=2x\hat{i}+4y\hat{j}+8z\hat{k}$ is conservative.

(e) Find the parametric representation of a circle having centre at the origin and radius r. Consider arc length as the parameter.

(f) The velocity $\vec{v}$ of a fluid in motion is given by $\vec{v} = [2y^2, 0, 0]$. Find whether the flow is

(i) irrotational

(ii) incompressible.

(g) Using Green's theorem show that the area A of the plane region bounded by a curve C is given by $A=\oint_C (xdy-ydx)$.

(h) Find the parametric representation of the Ellipsoid $x^2 + y^2 + \frac{1}{4}z^2 = 1$.

(i) Show that one can rewrite the formula of the divergence theorem as

$$\iiint_T \nabla^2 f\, dV = \iint_S \frac{\partial f}{\partial n} dA, \text{ I}$$

T is a closed bounded region in space whose boundary is piecewise smooth orientable surface S, grad f = $\vec{F}$, $\vec{F}(x, y, z)$ is a vector point function that has continuous first order partial derivatives in some domain T, grad f = $\vec{F}$ and $\hat{n}$ is the unit normal vector of S.

(j) Find $\int_0^{\pi/2} \cos^9 \theta\, d\theta$.

2. Solve using Laplace Transform : 7+3

(a) $y_1' = -y_2 + 1 - u(t-1),\ y_2' = y_1 + 1 - u(t-1),\ y_1(0) = 0,\ y_2(0) = 0.$

(b) Find $L^{-1}\left(\ln\left(1 + \frac{\omega^2}{s}\right)\right)$.

3. (a) Find $L^{-1}\left(\frac{s^2}{s^4 + 4a^4}\right)$. 5

(b) Using the Fourier Integral, evaluate : 5

$$\int_0^\infty \frac{\omega^3 \sin x\omega}{x^4 + 4} dx.$$

4. (a) Let f be a differentiable scalar function that represents a surface S : 5

f (x, y, z) = c = const.

If the gradient of f at a point P of S is not the zero vector, show that it is a normal vector of S at P.

(b) Show that : 5

$$\text{div}(f\,\vec{v}) = f \,\text{div}\, \vec{v} + \vec{v}.\nabla f.$$

5. (a) Let $f(x,y,z) = c/r$, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\vec{p} = -\frac{c}{r^3}\vec{r}$ and C is a constant.

Show that : 4+3

(i) grad $f = \vec{p}$

(ii) div(grad f) = 0.

(b) Show that curl (grad g) = 0, where g is a twice continuously differentiable scalar function. 3

6. (a) Evaluate : 5

$$\int_{(0,2,3)}^{(1,1,1)} (yz \sinh xz\, dx + \cosh xz\, dy + yx \sinh xz\, dz).$$

(b) Let I_x, I_y, I_o denote the moment of inertia of a mass in R about x-axis, y-axis and origin respectively. Find I_x, I_y, I_o of a mass of density $f(x, y) = 1$ in a triangular region in xy-plane with vertices at (0, 0), (b/2, h) and (b, 0). 5

7. (a) Evaluate the surface integral : 4

$$\iint_S G(\vec{r})\,dA,$$

for the given data $G = ye^{-xy}$, $S : z = 3x + 4y$, $x \geq 1$, $y \geq 1$.

(b) Evaluate : 3

$\iint_S \vec{F}.\vec{n}\, dA$, for the following data

$\vec{F} = [\cos y, \sin x, \cos z]$, S is the surface of $x^2 + y^2 \leq 4$, $|z| \leq 2$.

(c) Evaluate the line Integral $\int_C \vec{F}.\vec{r}(s)\,ds$ by Stokes's Theorem where C is the circle $x^2 + y^2 = 4$, $z = 1$ and $\vec{F} = [5y, 4x, z]$. 3

8. (a) Show that : 5

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(b+cx)^{m+n}}\,dx = \frac{1}{b^n}\frac{1}{(b+c)^m}\,\beta(m,n)$$

(b) Find the Fourier Series of $f(x)$ given by 5

$$f(x) = \begin{cases} x^2 & \text{if } -\pi/2 < x < \pi/2 \\ \pi^2/4 & \text{if } \pi/2 < x < 3\pi/2 \end{cases}$$

Assume that $f(x)$ has period 2π.