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Total number of printed pages – 4

B.Tech BS 1104

Second Semester Examination – 2012 MATHEMATICS – II

Full Marks - 70

Time: 3 Hours

Answer Question No. 1 which is compulsory and any five from the rest.

The figures in the right-hand margin indicate marks.

1. Answer the following questions:

2×10

- (a) Give example of a function that has no Laplace Transform. Indicate the reason.
- (b) If L(y(t)) = Y(s), find $L\left\{\int_0^t y(\tau)\cos(t-\tau)d\tau\right\}$.
- (c) Temperature at a point (x, y, z) in space is given by f(x, y, z) = xyz. If particle P at (-1, 1, 3) is to be moved to (0, -1, 5) on a straight line path, find the rate of change of temperature experienced by P.
- (d) When a vector field is said to be conservative? Show whether the vector function $\vec{v}(x,y,z)=2x\hat{i}+4y\hat{j}+8z\hat{k}$ is conservative.
- (e) Find the parametric representation of a circle having centre at the origin and radius r. Consider arc length as the parameter.
- (f) The velocity \vec{v} of a fluid in motion is given by $\vec{v} = [2y^2, 0, 0]$. Find whether the flow is
 - (i) irrotational
 - (ii) incompressible.
- (g) Using Green's theorem show that the area A of the plane region bounded by a curve C is given by $A = \oint_{\Gamma} (xdy ydx)$.

- (h) Find the parametric representation of the Ellipsoid $x^2 + y^2 + \frac{1}{4}z^2 = 1$.
- (i) Show that one can rewrite the formula of the divergence theorem as

$$\iiint_{T} \nabla^{2} f dV = \iint_{S} \frac{\partial f}{\partial n} dA, I$$

T is a closed bounded region in space whose boundary is piecewise smooth orientable surface S, grad $f = \vec{F}$, $\vec{F}(x, y, z)$ is a vector point function that has continuous first order partial derivatives in some domain T, grad $f = \vec{F}$ and \hat{n} is the unit normal vector of S.

- (j) Find $\int_0^{\pi/2} \cos^9 \theta \, d\theta$.
- 2. Solve using Laplace Transform:

7+3

- (a) $y_1' = -y_2 + 1 u(t-1)$, $y_2' = y_1 + 1 u(t-1)$, $y_1(0) = 0$, $y_2(0) = 0$.
- (b) Find $L^{-1}\left(\ln\left(1+\frac{\omega^2}{s}\right)\right)$.

3. (a) Find
$$L^{-1}\left(\frac{s^2}{s^4 + 4a^4}\right)$$
.

(b) Using the Fourier Integral, evaluate:

5

$$\int_0^\infty \frac{\omega^3 \sin x \omega}{x^4 + 4} \, dx.$$

4. (a) Let f be a differentiable scalar function that represents a surface S:

$$f(x, y, z) = c = const.$$

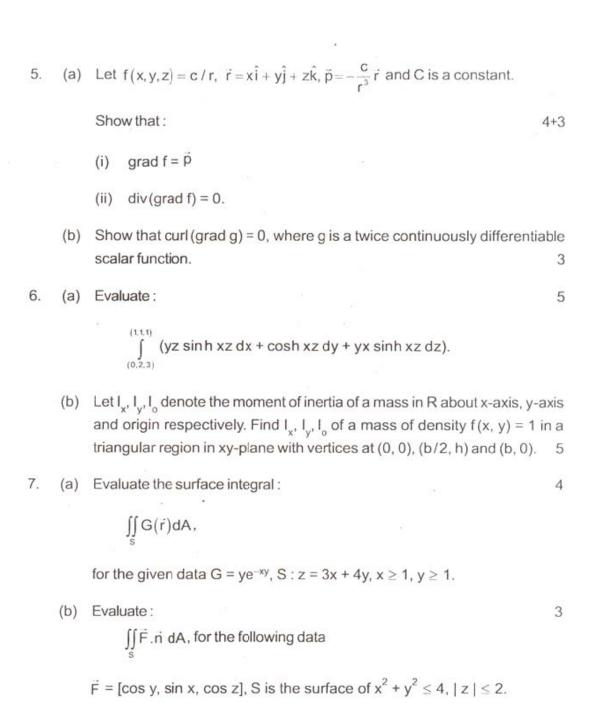
If the gradient of f at a point P of S is not the zero vector, show that it is a normal vector of S at P.

(b) Show that:

5

$$\operatorname{div}(f\vec{v}) = f\operatorname{div}\vec{v} + \vec{v} \cdot \nabla f$$
.

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(c) Evaluate the line Integral $\int_{C} \vec{F} \cdot \vec{r}(s) ds$ by Stokes's Theorem where C is the

circle
$$x^2 + y^2 = 4$$
, $z = 1$ and $F = [5y, 4x, z]$.

5

8. (a) Show that: 5

$$\int\limits_{0}^{1} \ \frac{x^{m-1} {\left(1-x\right)^{n-1}}}{\left(b+cx\right)^{m+n}} \ dx = \frac{1}{b^{n}} \, \frac{1}{\left(b+c\right)^{m}} \ \beta \left(m,n\right)$$

(b) Find the Fourier Series of f(x) given by

$$f(x) = \begin{cases} x^2 & \text{if } -\pi/2 < x < \pi/2 \\ \pi^2/4 & \text{if } \pi/2 < x < 3\pi/2 \end{cases}$$

Assume that f(x) has period 2π .