

Registration No. :

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Total number of printed pages – 2

B. Tech  
BSCM 2102 (Old)

## Second Semester (Back) Examination – 2013

### MATHEMATICS – II

BRANCH : ALL

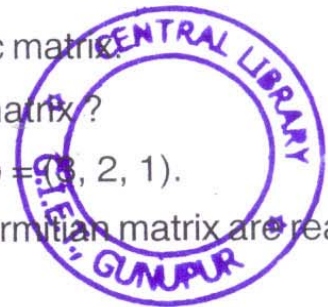
QUESTION CODE : B490

Full Marks – 70

Time : 3 Hours

Answer Question No. 1 which is compulsory and any **five** from the rest.  
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10
- (a) What is the condition for existence of a solution for a system of linear equations ?
  - (b) Define eigen value and eigen vector of a matrix.
  - (c) Check whether the vectors  $(2, 5, 0)$ ,  $(5, 3, 8)$ ,  $(8, 6, 3)$  in  $\mathbf{R}^3$  are linear independent or not.
  - (d) Define symmetric matrix, skew symmetric matrix.
  - (e) What do you mean by eigenspace of a matrix ?
  - (f) Find the projection of  $a = (5, 4, 2)$  over  $b = (3, 2, 1)$ .
  - (g) Prove that the diagonal elements of a Hermitian matrix are real.
  - (h) Prove that  $\text{curl}(\text{grad}f) = 0$ .
  - (i) Evaluate  $\int_0^3 \int_1^x (\sin xy + 5) dx dy$ .
  - (j) State Stokes' theorem.
2. (a) Solve the system of equations : 5
- $$\begin{aligned}x - y + z &= 0 \\ -x + y + -z &= 0 \\ 10y + 25z &= 90 \\ 20x + 10y &= 80\end{aligned}$$



- (b) Diagonalize the matrix  $\begin{bmatrix} 1 & 2 & -9 \\ -2 & -4 & 19 \\ 0 & -1 & 2 \end{bmatrix}$ . 5
3. (a) Find the eigenvalue and the eigen vectors of the matrix  $\begin{bmatrix} 1 & 3 & -5 \\ -6 & 13 & 3 \\ -8 & 13 & -5 \end{bmatrix}$ . 5
- (b) Find the inverse of a matrix using Gauss Jordan method  $\begin{bmatrix} 3 & 0 & -12 \\ -6 & 3 & 0 \\ 9 & 6 & 3 \end{bmatrix}$ . 5
4. (a) Find the directional derivative of  $x^2y^2z^2$  at the point  $(1, 1, -1)$  in the direction of the tangent to the curve  $(x = e^t, y = \sin 2t + 1, z = 1 - \cos t)$  at  $t = 0$ . 5
- (b) A vector field is given by  $\mathbf{A} = (x^2 + xy^2)\mathbf{i} + (x^2y + y^2)\mathbf{j}$ . Show that the field is irrotational and find the scalar potential. 5
5. (a) Determine whether the line integral  $\int 2xyz^2 dx + (x^2z^2 + z \cos yz) dy + (2x^2yz + y \cos yz) dz$  is independent of the path of integration. If so, then evaluate it from  $(1, 0, 1)$  to  $(0, \pi/2, 1)$ . 5
- (b) Evaluate  $\iint_S (yzi + xzj + xyk) \cdot ds$ , where  $S$  is the surface of the sphere  $x^2 + y^2 + z^2 = a^2$  in the first octant. 5
6. (a) State and verify Green's theorem in the plane for  $\oint_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$  where  $C$  is the boundary region bounded by  $x \geq 0, y \leq 0$  and  $2x - 3y = 6$ . 5
- (b) Evaluate  $\iint_S \mathbf{f} \cdot \mathbf{n} ds$  where  $\mathbf{f} = y^2\mathbf{i} + y\mathbf{j} + xz\mathbf{k}$  and  $S$  is the upper half of the sphere  $x^2 + y^2 + z^2 = 9, z > 0$ . 5
7. (a) Find the Fourier series expansion of  $f(x) = x^3, 0 < x < 2\pi$ . 5
- (b) Find the Fourier sine integral representation of  $f(x) = \sin x$ , if  $0 < x < \pi$  and 0 if  $x > \pi$ . 5
8. (a) Find Fourier cosine transform of  $f(x) = e^{-x}$ . 5
- (b) Express the integral  $\int_0^1 x^m (1 - x^n)^p dx$  in terms of gamma function and hence evaluate  $\int_0^1 x^2 (1 - x^2)^4 dx$ . 5