Registration No. :

Total number of printed pages – 2

B. Tech

BSCM 2102 (Old)

Second Semester (Back) Examination – 2013

MATHEMATICS – II

BRANCH : ALL

QUESTION CODE : B490

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

   (a) What is the condition for existence of a solution for a system of linear equations ?

   (b) Define eigen value and eigen vector of a matrix.

   (c) Check whether the vectors (2, 5, 0), (5, 3, 8), (8, 6, 3) in $\mathbf{R}^3$ are linear independent or not.

   (d) Define symmetric matrix, skew symmetric matrix.

   (e) What do you mean by eigenspace of a matrix ?

   (f) Find the projection of a = (5, 4, 2) over b = (3, 2, 1).

   (g) Prove that the diagonal elements of a Hermitian matrix are real.

   (h) Prove that curl (gradf) = 0.

   (i) Evaluate $\int_0^3 \int_1^x (\sin xy + 5)\, dxdy$.

   (j) State Stokes' theorm.

2. (a) Solve the system of equations : 5

$$x - y + z = 0$$

$$-x + y + -z = 0$$

$$10y + 25z = 90$$

$$20x + 10y = 80$$

P.T.O.

(b) Diagonalize the matrix $\begin{bmatrix} 1 & 2 & -9 \\ -2 & -4 & 19 \\ 0 & -1 & 2 \end{bmatrix}$. 5

3. (a) Find the eigenvalue and the eigen vectors of the matrix $\begin{bmatrix} 1 & 3 & -5 \\ -6 & 13 & 3 \\ -8 & 13 & -5 \end{bmatrix}$. 5

(b) Find the inverse of a matrix using Gauss Jordan method $\begin{bmatrix} 3 & 0 & -12 \\ -6 & 3 & 0 \\ 9 & 6 & 3 \end{bmatrix}$. 5

4. (a) Find the directional derivative of $x^2y^2z^2$ at the point (1, 1, –1) in the direction of the tangent to the curve ($x = e^t$, $y = \sin 2t + 1$, $z = 1 - \cos t$) at $t = 0$. 5

(b) A vector field is given by $\mathbf{A} = (x^2 + xy^2)i + (x^2y + y^2)j$. Show that the field is irrotational and find the scalar potential. 5

5. (a) Determine whether the line integral $\int 2xyz^2 dx + (x^2z^2 + z\cos yz)dy + (2x^2yz + y\cos yz)dz$ is independent of the path of integration. If so, then evaluate it from (1, 0, 1) to (0, $\pi/2$, 1). 5

(b) Evaluate $\iint_s (yzi + zxj + xyk).ds$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. 5

6. (a) State and verify Green's theorem in the plane for $\oint (3x^2 - 8y^2)dx + (4y - 6xy)\,dy$ where C is the boundary region bounded by $x \geq 0$, $y \leq 0$ and $2x - 3y = 6$. 5

(b) Evaluate $\iint_s f \bullet nds$ where $f = y^2 i + yj + xzk$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = 9$, $z > 0$. 5

7. (a) Find the Fourier series expansion of $f(x) = x^3$, $0 < x < 2\pi$. 5

(b) Find the Fourier sine integral representation of $f(x) = \sin x$, if $0 < x < \pi$ and 0 if $x > \pi$. 5

8. (a) Find Fourier cosine transform of $f(x) = e^{-x}$. 5

(b) Express the integral $\int_0^1 x^m(1 - x^n)^p dx$ in terms of gamma function and hence evaluate $\int_0^1 x^2(1 - x^2)^4 dx$. 5
