Registration No. : | | | | | | | | | | |

Total number of printed pages – 3

B. Tech
BS 1104

Second Semester Examination – 2013

MATHEMATICS – II

QUESTION CODE : A438

Full Marks – 70

Time : 3 Hours

*Answer Question No. 1 which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) Define convolution of two functions.

(b) Find Inverse Laplace Transform of $\frac{e^{-5s}}{s^2+9}$.

(c) Find the Laplace Transform of $L\left\{\int_0^5 \sin x dx\right\}$.

(d) Prove that $\beta(m, n) = \beta(n, m)$.

(e) Define error function.

(f) Write the formula to find the length of a curve from origin to a point having position vector **a**.

(g) Find the area of a cardoid $r = 1 - \cos\theta,\ 0 \le \theta \le 2\pi$.

(h) What is the geometrical significance of curl of a vector ?

(i) What is directional derivative ? What is the directional derivative in the direction of any non-unit vector **a** ?

(j) State Gauss divergence theorem.

P.T.O.

2. (a) Using convolution theorem find $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}, a \neq b.$ 5

(b) Solve the simultaneous differential equation using Laplace transform 5

$y_1^1 - y_2 = e^t$

$y_2^1 + y_1 = \sin t, \quad y_1(0) = 1, \; y_2(0) = 0.$

3. (a) Obtain a Fourier series to represent $\cos ax$ in $(-\pi, \pi)$, where a is an integer. 5

(b) Find the Fourier sine series expansion of $f(x) = x, \quad 0 < x < \pi/2$ 5

$= \pi - x, \quad \pi/2 < x < \pi.$

4. (a) Using Fourier Integral representation, show that $e^{-ax} = \frac{2a}{\pi}\int_0^\infty \frac{\cos\lambda x}{\lambda^2 + a^2} d\lambda$ 5

(b) Find the Fourier transform of $f(x) = \cos x$ when $-a \le x \le a$ 5

$= 0$ otherwise.

5. (a) Find the directional derivative of $x^2y^2z^2$ at the point (1,1,–1) in the direction of the tangent to the curve $x = e^t, y = \sin 2t + 1, z = 1 - \cos t$ at $t = 0$. 5

(b) Prove that grad $(\mathbf{f}.\mathbf{g}) = \mathbf{f} \times \text{curl}\,\mathbf{g} + \mathbf{g} \times \text{curl}\,\mathbf{f} + (\mathbf{f}.\Delta)\,\mathbf{g} + (\mathbf{g}.\Delta)\,\mathbf{f}$. 5

6. (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2). 5

(b) A vector field is given by $A = (x^2 + xy^2)\,i + (x^2y + y^2)j$. Show that the field is irrotational and find the scalar potential. 5

7. (a) Determine whether the line integral $\int 2xyz^2dx + (x^2z^2 + z\cos yz)\,dy + (2x^2yz + y\cos yz)dz$ is independent of the path of integration. If so, then evaluate it from $(1, 0, 1)$ to $(0, \pi/2, 1)$. 5

(b) Evaluate $\iint_S (yzi + zxj + xyk).ds$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. 5

8. (a) State and verify Green's theorem in the plane for $\oint (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary region bounded by $x \geq 0$, $y \leq 0$ and $2x - 3y = 6$. 5

(b) Verify Stokes' theorem for the vector field $\mathbf{F} = (2x - y)\,i - yz^2\,\mathbf{j} - y^2z\,\mathbf{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = 1$ bounded by its projection on xy-plane. 5

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