Registration No. :											
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Total number of printed pages - 3

B. Tech BS 1104

Second Semester Examination - 2013

MATHEMATICS - II

QUESTION CODE: A438

Full Marks - 70

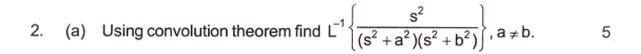
Time: 3 Hours

Answer Question No. 1 which is compulsory and any five from the rest.

The figures in the right-hand margin indicate marks.

- 1. Answer the following questions:
 - (a) Define convolution of two functions.
 - (b) Find Inverse Laplace Transform of $\frac{e^{-5s}}{s^2 + 9}$.
 - (c) Find the Laplace Transform of L $\begin{cases} \int_0^5 \sin x dx \\ \end{bmatrix}.$
 - (d) Prove that β (m, n) = β (n, m).
 - (e) Define error function.
 - (f) Write the formula to find the length of a curve from origin to a point having position vector **a**.
 - (g) Find the area of a cardoid $r = 1 \cos \theta$, $0 \le \theta \le 2\pi$.
 - (h) What is the geometrical significance of curl of a vector?
 - (i) What is directional derivative? What is the directional derivative in the direction of any non-unit vector **a**?
 - (j) State Gauss divergence theorem.

2×10



(b) Solve the simultaneous differential equation using Laplace transform 5

$$y_1^1 - y_2 = e^t$$

 $y_2^1 + y_1 = sint, y_1(0) = 1, y_2(0) = 0.$

- 3. (a) Obtain a Fourier series to represent $\cos x$ in $(-\pi, \pi)$, where π is an integer.
 - (b) Find the Fourier sine series expansion of f(x) = x, $0 < x < \pi/2$ 5 $= \pi x, \quad \pi/2 < x < \pi.$
- 4. (a) Using Fourier Integral representation, show that $e^{-ax} = \frac{2a}{\pi} \int_{0}^{\infty} \frac{\cos \lambda x}{\lambda^2 + a^2} d\lambda$ 5
 - (b) Find the Fourier transform of $f(x) = \cos x$ when $-a \le x \le a$ 5 = 0 otherwise.
- 5. (a) Find the directional derivative of $x^2y^2z^2$ at the point (1,1,-1) in the direction of the tangent to the curve $x = e^t$, $y = \sin 2t + 1$, $z = 1 \cos t$ at t = 0.
 - (b) Prove that grad $(\mathbf{f}.\mathbf{g}) = \mathbf{f} \times \text{curl } \mathbf{g} + \mathbf{g} \times \text{curl } \mathbf{f} + (\mathbf{f}.\Delta) \mathbf{g} + (\mathbf{g}.\Delta) \mathbf{f}$. 5
- 6. (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 3$ at the point (2, -1, 2).
 - (b) A vector field is given by $A = (x^2 + xy^2)i + (x^2y + y^2)j$. Show that the field is irrotational and find the scalar potential.

- 7. (a) Determine whether the line integral $\int 2xyz^2dx + (x^2z^2 + z\cos yz)dy + (2x^2yz + y\cos yz)dz$ is independent of the path of integration. If so, then evaluate it from (1, 0, 1) to (0, $\pi/2$, 1).
 - (b) Evaluate $\iint_s (yzi + zxj + xyk).ds$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant.
- 8. (a) State and verify Green's theorem in the plane for $\oint (3x^2 8y^2)dx + (4y 6xy)dy$ where C is the boundary region bounded by $x \ge 0$, $y \le 0$ and 2x - 3y = 6.
 - (b) Verify Stokes' theorem for the vector field $\mathbf{F} = (2x y)\mathbf{i} yz^2\mathbf{j} y^2z\mathbf{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = 1$ bounded by its projection on xy-plane.