Registration No. : | | | | | | | | | | |

Total number of printed pages – 3

B. Tech
BS 1104

Second Semester Regular Examination – 2015

MATHEMATICS – II

BRANCH : AEIE, AUTO, BIOMED, BIOTECH, CHEM, CIVIL, CSE, EC, EEE, EIE, ELECTRICAL, ETC, FASHION, IT, MANUTECH, MECH, MINERAL, MINING, MM, MME, PLASTIC, TEXTILE

QUESTION CODE : J 217

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) Write the existence theorem of Laplace equation.

(b) Find the Inverse Laplace transform of $\frac{3w}{s^2\left(s^2+9w^2\right)}$

(c) Find the Laplace transform of an t- step function.

(d) Explain convergence of a Fourier series.

(e) What is the importance of half range series ?

(f) What is the physical significance of divergence, how it is related to the Laplacian ?

(g) Find $\Delta^2 f$,where $f = e^{x^2-y^2}\cos 2xy$

(h) Prove that div(curl v) = 0

P.T.O.

(i) Evaluate $\iint_C (2\sin xy + 3y)\,dxdy$ along the straight line $x + y = 1$.

(j) State Stoke's theorem

2. (a) Solve the integral equation $y(t) = 1 - \int_0^t (t - x)\,y(x)\,dx$ using Laplace transform. 5

(b) Using Laplace equation , solve the differential equation
$y'' - 5y' + 6y = 4e^t$,$0 < t < 2$, and 0 if $t \geq 2$ with $y(0) = 1$,$y'(0) = -2$. 5

3. (a) Solve the equation by the transform method : 5

$$\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t,\ y(0) = 0,\ y'(0) = 1$$

(b) Find the Fourier sine series expansion of 5
$f(x) = x,\ 0 < x > \pi$ and $\pi - x,\ \pi < x < 2\pi$

4. (a) Using Fourier integral representation, show that 5

$$\int_0^\infty \frac{\cos wx + w \sin wx}{1 + w^2}\,dw = \begin{cases} 0, & \text{if } x < 0 \\ \pi/2, & \text{if } x = 0 \\ \pi e^{-x}, & \text{if } x > 0 \end{cases}$$

(b) Find the Fourier series expansion of $f(x) = \pi \sin \pi x,\ 0 < x \leq 1$. 5

5. (a) Find the Fourier sine transform of $f(x) = e^{-|x|}$, hence evaluate 5

$$\int_0^\infty \frac{x \sin mx}{1 + x^2}\,dx$$

(b) Find the acute angle between the surfaces
$xyz^2 = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point $(1, -2, 1)$. 5

6. (a) Find the directional derivative of $xyz^2 + xz$ at $(1,1,1)$ in the direction of the normal to the surface $3xy^2 + y + z$ at $(0,1,1)$ 5

(b) Find the scalar potential of $f = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$. 5

7. (a) If $F = [y-2x, 3x + 2y]$ is velocity of a fluid, then find the circulation of 'F' around a circle in the xy-plane with centre at origin and radius 5. 5

(b) Determine whether the line integral 5

$\int 2xyz^2dx + (x^2y^2 + z \cos yz)dy + (2x^2 yz + y \cos yz)\, dz$ is independent of the path of integration. If so, then evaluate it from $(1,0,1)$ to $(0, \Pi/2, 1)$.

8. (a) Find the surface integral $\iint F \cdot ndA$, where $F = [x^2, e^y, 1]$, 5

$S : x + y + z = 1, x \geq 0, y \geq 0, z \geq 0$.

(b) Verify Gauss divergence theorem for a vector field defined by 5

$F = [y, x, z^2]$ the cylindrical region S given by $x^2 + y^2 = a^2$; $z = 0$ and $z = h$.