Registration No. : | | | | | | | | | | |

Total number of printed pages – 3

B. Tech
BS 1104

Second Semester Back Examination – 2015

MATHEMATICS – II

BRANCH (S) : AEIE, AERO, AUTO, BIOMED, BIOTECH, CHEM, CIVIL, CSE, EC, EEE, EIE, ELECTRICAL, ENV, ETC, FASHION, FAT, IEE, IT, MANUFACT, MANUTECH, MECH, MINERAL, MINING, MM, MME, PLASTIC, TEXTILE

QUESTION CODE : M 170

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) Define Laplace transform of any function.

(b) Find the Inverse Laplace transform of $\frac{e^{-2s}}{(s^2+9)}$.

(c) State convolution theorem. How is it helpful ?

(d) Find the smallest positive period of $\cos(n\pi x/L)$ and $\tan(n+1)x$.

(e) What is half range series ?

(f) What is the physical significance of curl ?

(g) Find the projection of vector $\mathbf{a} = (5,4,2)$ over vector $\mathbf{b} = (3,2,1)$.

(h) Define exactness of an integral.

P.T.O.

(i) Evaluate $\int_C \int (2 \sin xy + 3y)\, dxdy$ along the straight line $x^2 + y = 1$.

(j) State Green's theorem.

2. (a) Find the Laplace transform of 5

(i) $f(t) = e^{2t} * \cosh 3t$

(ii) $f(t) = (t-1)^2 u(t-2)$

(b) Using Laplace transform, solve the differential equation 5

$y'' - 2y' - 8y = 5\sin 3t$ when $y(0) = 1$, $y'(0) = -2$.

3. (a) Solve the integral equation, $y(t) = 2t - \int_0^t (t-x)\, y(x)\, dx$ using Laplace transform. 5

(b) Find the Fourier series expansion of $f(x) = e^{2x}$, $-\pi < x < \pi$. 5

4. (a) Find the fourier cosine series expansion of $f(x) = 3x(\pi^2 - x^2)$ in the range $-\pi < x < \pi$. 5

(b) Using Fourier integral representation, show that 5

$$\int_0^\infty \frac{w \sin wx}{1+w^2}\, dw = \begin{cases} 0, & \text{if } x<0 \\ \pi/2, & \text{if } x=0 \\ \pi e^{-x}, & \text{if } x>0 \end{cases}$$

5. (a) Find the Fourier sine transform of $f(x) = e^{-|x|}$, hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2}\, dx$ 5

(b) Determine the value of "a" such that the vector $\mathbf{V} = (ax^2y + yz)\,\mathbf{i} + (xy^2 + xz^2)\,\mathbf{j} + (2xyz - 2x^2y^2)\,\mathbf{k}$ has zero divergence. 5

6. (a) Find the directional derivative of $f(x,y,z) = 5e^{2x - y + z}$ at $(1,1,-1)$ in the direction of the normal to the surface $3xy^2 + y + z$ at $(0,1,1)$ 5

(b) Find the scalar potential of $\mathbf{f} = (6xy + z^3)\,\mathbf{i} + (3x^2 - z)\,\mathbf{j} + (3xz^2 - y)\,\mathbf{k}$. 5

7. (a) If $F = [y - 2x, 3x + 2y]$ is velocity of a fluid, then find the circulation of 'F' around a circle in the xy-plane with centre at origin and radius 5. 5

(b) Evaluate the line integral where $\int_C f.dr$ where $f = (2y + 3)\mathbf{i} + xz\,\mathbf{j} + (yz - x)\mathbf{k}$ along the straight line joining (0,0,0) and (2,1,1). 5

8. Find the surface integral $\iint F.ndA$, where $F = [x^2, e^y, 1]$, 10

$S : x + y + z = 1, x \geq 0, y \geq 0, z \geq 0.$