Registration No:

Total Number of Pages: 02 **B.TECH**
15BS1104

2nd Semester Back Examination 2016-17
MATHEMATICS- II
BRANCH: ALL
Time: 3 Hours
Max Marks: 100
Q.CODE: Z338
Answer Part-A which is compulsory and any four from Part-B.
The figures in the right hand margin indicate marks.

Part – A (Answer all the questions)

Q1 Answer the following questions: ***multiple type or dash fill up type*** **(2 x 10)**

- **a)** Find $L^{-1}[\frac{s+3}{(s-2)(s+1)}]$
- **b)** Find L$(e^{-2t}\sin at)$.
- **c)** Write the fundamental period of f(x)= $\cos\frac{5}{7}\pi x$.
- **d)** Using Gamma function find the value of $\int_0^\infty x^5 e^{-x} dx$.
- **e)** Prove that $\nabla\cdot(\nabla f) = \nabla^2 f$.
- **f)** Find the Fourier sine transformation of the function f(x)= e^{-2x}
- **g)** Evaluate $\int_C F(r)\cdot dr, where\ F = [y^2, -x^2]$ and C: Be the line segment from (0, 0) to (1,1)?
- **h)** Using Beta function find the value of $\beta(3,2)$.
- **i)** Find the value of $1 * x$
- **j)** State Gauss divergence theorem?

Q2 Answer the following questions: ***Short answer type*** **(2 x 10)**

- **a)** Find the Laplace transformation of the function f(t) = $\frac{\sin \alpha t}{t}$
- **b)** Find $\nabla^2 f\ where\ f = x^2 + y^2 + z^2$
- **c)** Find theLaplace transformationofthe function $f(t) = (2^t)$
- **d)** Find the Directional derivative of the function f $= x - y$ at a point p(4,5) in the direction $\vec{a} = 2\hat{\imath} + \hat{\jmath}$
- **e)** Determine the constant 'b' such that f$(x, y, z) = [bx^2y + yz,\ xy^2 - xz^2,$ $2xyz - 2x^2y^2]$ has divergence zero.
- **f)** Find the Laplace transformation of the unit impulse function $\delta(t - 2^{2015})\ and$ The unit step function $U(t - 2^{2015})$.
- **g)** Find the Fourier cosine series of the function f(x)= $-1(-\pi < x < 0)$; f(x)= $1(0 < x < \pi)$.
- **h)** Find a parametric representation of the elliptic cylinder $9x^2 + 4y^2 = 36$.
- **i)** Find L[f(t)], Where $f(t) = \begin{cases} 5; 0 < t < 1 \\ 6; 1 < t < 4 \\ 0; \quad t > 4 \end{cases}$

j) Find the value of $L^{-1}\left[\frac{1}{s^2(s^2+1)}\right]$ using convolution theorem.

Part – B (Answer any four questions)

Q3 a) Solve the following initial value problem using Laplace transformation **(10)**

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4e^{2t} \; with \; y(0) = -3, y'(0) = 5$$

b) Show that $L(\sin\sqrt{t}) = \frac{1}{2s}\sqrt{\frac{\pi}{s}}e^{(-1/4s)}$ **(5)**

Q4 a) Verify Green's Theorem in the plane for **(10)**
$\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, Where 'C' is the closed curve of the region bounded by y= $x^2 \; and \; y = \sqrt{x}$

b) Find the area of the region in the first quadrant under the arc of the Cardioid r = $5(1 - \cos\theta)$; $0 \leq \theta \leq 2\pi$ **(5)**

Q5 a) Prove that the integral $\int_0^\infty \frac{\omega \sin \omega x + \cos \omega x}{1+\omega^2} d\omega = \begin{cases} 0 & ; x < 0 \\ \frac{\pi}{2} & ; x = 0 \\ \pi e^{-x} & ; \; x > 0 \end{cases}$ **(10)**

b) Prove that $\Gamma(n+1) = n\,\boldsymbol{\Gamma} n\,, n > 0$. **(5)**

Q6 a) Solve the following integral equation using Laplace transformation **(10)**
$y(t) = 1 + \int_0^t \sin(t-u)y(u)du$.

b) Using convolution prove that **(5)**
$1 * 1 * 1 * \cdots\cdots\cdots * 1(upto \; 'K' \; times) = \frac{t^{K-1}}{(K-1)!}$

Q7 a) Find the moment of inertia $I_x \; and \; I_y$ about X$-axes \; and$ Y- axes respectively and also find polar moment of inertia I_o of a mass of density $f(x, y) = 1$ in the region **(10)**

R: $0 \leq x \leq 1, 0 \leq y \leq \sqrt{1-x^2}$

b) Find the total Mass of a mass distribution of density f$(x, y, z) = x^2 + y^2 + z^2$ in a region T: $-1 \leq x \leq 1, -3 \leq y \leq 3, -2 \leq z \leq 2$ **(5)**

Q8 a) Verify Divergence Theorem for F= $4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ taken over the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0 \; and \; z = 1$. **(10)**

b) Find the coordinates of the center of gravity of a mass of density f$(x, y) = 1$ in the region R : the region $x^2 + y^2 \leq 4$ **(5)**

Q9 a) Find the Fourier Transformation of $f(x) = \begin{cases} 0 \;, & x > 0 \\ e^{2x}, & x < 0 \end{cases}$ **(10)**

b) Find the Fourier series expansion of the function **(5)**

$$f(x) = \frac{\pi - x}{2} (0 < x < 2)$$