Registration no: | | | | | | | | | | |

Total Number of Pages: 2 **B.Tech**
BS1104

2^{nd} Semester Back Examination 2016-17
MATHEMATICS - II
BRANCH:ALL
Time: 3 Hours
Max Marks: 70
Q.CODE: Z339

Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.

Q1 **Answer the following questions:** **(2 x 10)**

- **a)** Find $1 * 1 * 1$
- **b)** Find Laplace transformation of $f(t) = t^2 e^{at}$
- **c)** Find $L^{-1}[\frac{1}{(s-10)(s-20)}]$
- **d)** Using Beta function find the value of $\beta(2,2)$
- **e)** Let $f(x,y,z) = 3x^2y - z^2y^3$, find ∇f and $|\nabla f|$ at (1,1,1)
- **f)** Evaluate divergence of $f(x,y,z) = 2x^2z\hat{i} - xy^2z\hat{j} + 3yz^2\hat{k}$ at (1,1,1)
- **g)** What is the fundamental period of $f(x) = \sin 5\pi x$
- **h)** Evaluate $\int_C F(r) \cdot dr$, where $F = [xy, -x^2y^2]$ and C: Be the line segment from (2, 0) to (0, 2).
- **i)** Find the Fourier sine series of the function f(x)= $-k$ $(-\pi < x < 0)$; f(x)= k $(0 < x < \pi)$
- **j)** What is the parametric representation of the semicircle $x^2 + y^2 = 4$

Q2 **a)** Show that $\Gamma(n+1) = n!$ where 'n' is a nonnegative integer. **(2)**

b) Solve the following initial value problem using Laplace transformation **(8)**

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 1 \text{ with } y(0) = 0, y'(0) = 1$$

Q3 **a)** Prove that the integral $\int_0^\infty \frac{(1-\cos \pi\omega)\sin \omega x}{\omega} d\omega = \begin{cases} \frac{\pi}{2}, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ **(5)**

b) Find the Fourier series expansion of $f(x) = 2x$ $(-1 < x < 1)$ with $P = 2L = 2$. **(5)**

Q4 **a)** Find the area of the region in the first quadrant under the arc of the limacon r = 1 + 2 cos θ ; 0$\leq$ $\theta \leq \frac{\pi}{2}$ **(5)**

b) Using Greens theorem evaluates$\oint_C (x^2e^y)dx + y^2e^x dy$, where ‘C’ is the rectangle with vertices(0,0) , (2,0), (2,3) and (0,3). **(5)**

Q5 **a)** Find the moment of inertia I_x and I_y about X$-axes$ and Y- axes respectively of a mass of density $f(x, y) = 1$ in the region

R: 0$\leq x \leq 1, 0 \leq y \leq \sqrt{1 - x^2}$ **(5)**

b) Find the value of surface integral $\iint_S F \cdot n \, dA$,Where F=[$3x^2$, y^2 ,0] And the surface S: r= $[u \ , \ v \ , \ 2u + 3v]$, 0$\leq u \leq 2, -1 \leq v \leq 1$ **(5)**

Q6 **a)** Evaluate $\int_S (ax\hat{\imath} + by\hat{\jmath} + cz\hat{k}) \cdot n \, dS = \frac{4}{3}\pi(a + b + c)$where ‘S’ Is the surface of the sphere$x^2 + y^2 + z^2 = 1$ **(5)**

b) If F= $zy \, \hat{\imath} + xz \, \hat{\jmath} - xy \, \hat{k}$ then find $\int_C F \cdot dr$ Where ‘C’ is given by x= $t, y = t^2, z = t^3$ from P(0,0,0) to Q(2,4,8). **(5)**

Q7 Verify Divergence Theorem for F= $z\hat{\imath} + x\hat{\jmath} - yz\hat{k}$ taken over the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. **(10)**

Q8 **Write short answer on any TWO:** **(5 x 2)**

a) Find the Fourier Transformation of $f(x) = \begin{cases} e^x \ , & -a < x < a \\ 0, & elsewhere \end{cases}$

b) Find$L^{-1}\left[Log\left(\frac{s+1}{s-1}\right)\right]$

c) Solve the following integral equation using Laplace transformation

$$y(t) = \sin 2t + \int_0^t \sin 2(t - u)y(u)du$$

d) Find the Fourier series expansion of $f(x) = x \ (-\pi < x < \pi)$ with f$(x + 2\pi) = f(x)$