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Total number of printed pages – 4

B. Tech  
BSCM 2101/BS 1101(O/N)

**First Semester Examination – 2010**

**MATHEMATICS – I**  
(Old and New Course)

**Full Marks – 70**

**Time : 3 Hours**

*Answer Question No. 1 which is compulsory and any five from the rest.*

*The figures in the right-hand margin indicate marks.*

1. Answer the following questions precisely : 2 × 10
- (a) Find the solution of the differential equation  $y' - 2xy = 2x$ .
- (b) Find the solution of the initial value problem  $y' - e^x y = 0$  with  $y(0) = e$ .
- (c) Find the integrating factor of the differential equation  
 $(x + xy) dx + (y + xy) d(y) = 0$ .
- (d) Write the particular solution of the differential equation  $y'' + y' = 2x$  in general form using method of undetermined coefficient.
- (e) If  $y_1(x)$  and  $y_2(x)$  are solutions of the differential equation  $y'' + p(x)y' + q(x)y = 0$ , then what is the relation among  $y_1(x)$ ,  $y_2(x)$  and  $p(x)$ .
- (f) What is the radius of convergence of the power series  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$  ?

P.T.O.

(g) What necessary condition should  $p(x)$  satisfy in order to take

$$y(x) = \sum_{n=0}^{\infty} a_n x^{n+r} \text{ as the series solution of the differential equation } y'' + p(x)y' + y = 0$$

6.

(h) If  $J_n(x)$  is the Bessel function of order  $n$ , then what is the relation between  $J_n(x)$  and  $J_{-n}(x)$  ?

(i) If  $P_n(x)$  is the Legendre polynomial of degree  $n$ , then what is the relation between  $P_n(-x)$  and  $P_n(x)$  ?

(j) What is the relation between diagonal matrix of order  $n$  and identity matrix of order  $n$  ?

2. Solve the following problems :

(a) Find the radius of curvature of the curve  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$  at the point  $(0,a)$ . 5

(b) Find the asymptote to the curve  $x^3 + y^3 + 3axy = 0$ . 5

3. Answer the following questions according to the instruction :

(a) Solve the Bernoulli's equation  $x^3 y' + 4x^2 \tan(y) = e^x \sec(y)$ . 5

(b) Solve the non-linear differential equation  $(3x - 2y + 2) dx = (2x - 4y + 5) dy$ . 5

4. Solve the following initial value problems :

(a)  $y'' + y = \sin(x)$  with  $y(0) = 0$  and  $y'(0) = 2$  using method of undetermined coefficient. 5

(b)  $y'' + 2y' + y = xe^x$  with  $y(0) = 0$  and  $y'(0) = 0$  using method of variation of parameter. 5

5. Answer the following questions according to the instruction :

(a) Find the general solution of the differential equation  $y'' + y = \sin(x)$  using  $y(x) = \sin(x)$  as a solution of the homogeneous differential equation by the method of reduction order. 5

(b) Solve Cauchy-Euler equation  $x^2 y'' - x y' + y = \ln(x)$ . 5

6. Answer according to the instruction :

(a) Show that  $J_{\frac{1}{2}}(x) = \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \sin(x)$  5

(b) Show that  $\int_{-1}^1 P_n(x) P_m(x) dx = 0$  for  $m \neq n$ . 5

7. Answer according to the instruction :

(a) State the conditions under which a system of equations  $AX = b$  has unique solution, no solution and infinitely many solution. Find a solution of the system of equations in integer form 5

$$x + y + z = 3$$

$$2x + 3y + z = 4$$

$$2x + 2y + 2z = k$$

(b) What is the relation between the algebraic multiplicity and the geometric multiplicity of an eigenvalue of any square matrix? Find the eigenvalue and the corresponding eigenvectors of the matrix  $I_{3 \times 3}$ . 5

Or

(a) Find the Laplace transform of the function 5

$$f(t) = \begin{cases} 0, & t < 2 \\ 2t, & 2 < t < 4 \\ 0, & \text{otherwise} \end{cases}$$

using unit step function.

(b) If  $f(t) * g(t) = \int_0^t f(t-x)g(x) dx$ , then show that  $f(t) * g(t) = g(t) * f(t)$ . 5

8. Answer the following questions according to the instruction :

- (a) Find the name of the conic section which is represented by the quadratic form  $9x^2 + y^2 - 6xy = 40$ , and the corresponding transform which transforms the quadratic form to that conic section. 5
- (b) Find the orthogonal matrix  $P$  such that  $PAP^T$  is a diagonal matrix where the matrix  $A$  is 5

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Or

- (a) Solve the integral equation  $y(t) = e^t + \int_0^t y(x)e^{t-x} dx$  using Laplace transform. 5
- (b) Solve the initial value problem  $y'' + y = 2$  with  $y(0) = 0$  and  $y'(0) = 2$  using Laplace transform. 5