6. (*a*) Solve the following Cargo-Loading problem :

Item (n)	Weight (w_n)	Value (v_n)
1	2	7
2	3	10
3	1	3

$N = 5, N = 3$ and there is no volumes restriction.

Or

(*b*) A can is to be made in the form of a right circular cylinder to contain at least V cubic inches of oil. What diminsions of the can will require the least amount of material ?

(*c*) Minimize $g_0(x) = 2\pi x_1^2 + 2\pi x_1 x_2 + 2\pi x_1^{-1} x_2$ subject to

$$g_1(x) = 16 x_1^{-2} x_2^{-1} \le 1, \; x_1 > 0, \; x_2 > 0.$$

2018

Time : 3 hours

Full Marks : 80

Answer **all** sections as per direction

The figures in the right-hand margin indicate marks

Candidates are required to answer in their own words as far as practicable

Symbols used have their usual meaning

(OPTIMIZATION TECHNIQUES - II)

SECTION – A

1. Answer *all* questions : 2×8

(*a*) Define a general quadratic program.

(*b*) Define a signomial.

(*c*) What is a projectile ?

(*d*) State weak duality theorem.

(*e*) What is principle of optimality ?

(*Turn Over*)

(*f*) Define a dual quadratic program.

(*g*) What is a complementarity problem ?

(*h*) What is Frank-Wolfe method ?

Or

2. Answer any *four* questions : 4×4

(*a*) Show that quadratic program can not have an unbounded solution when G is a positive definite matrix.

(*b*) Let $p_i > 0, q_i > 0$ such that

$$\sum_{i=1}^{r} p_i = \sum_{i=1}^{r} q_i .$$

Then show that

$$\sum_{i=1}^{r} p_i \ln p_i \geq \sum_{i=1}^{r} p_i \ln q_i$$

(*c*) Find an expression for maximum height attained by projectile using dynamic program.

(*d*) Show that x° solves the problem I : Minimize

$f(x)$, subject to $g_i(x) \geq 0$ $(i = 1, 2, ..., q)$ where $x = (x_1, x_2, ..., x_p)^T$ if and only if $\begin{pmatrix} x^\circ \\ x^\circ_{p+1} \end{pmatrix} x^\circ_{p+1} = f(x^\circ)$ solve

II : Minimize $z = x_{p+1}$ subject to $x_{p+1} - f(x) \geq 0, g_i(x) \geq 0, i = 1, 2, ..., q$ where $x = (x_1, x_2, ..., x_p)^T$.

(*e*) A point $x^\circ \in \Omega_p$ is an optimal solution to program QP if and only if for some $\lambda^\circ \geq 0$, (x°, λ°) is a saddle point of the Lgarangian

$$L(x, \lambda) = f(x) \lambda^T (Ax - b),$$

where $\lambda \geq 0$.

(*f*) Explain the difference between Wolfe's method and Beale's method for solving a quadratic programming problem.

SECTION – B

Answer all questions : 16×4

3. (*a*) Explain Wolfe's algorithm for solving a quadratic programming problem.

Or

(*b*) Solve the following problem using Beale's method :

Minimize

$$f(x_1, x_2) = -10x_1 - 25x_2 + 10x_1^2 + 4x_1x_2 + x_2^2$$

subject to

$$x_1 + 2x_2 \leq 10;\ x_1 + x_2 \leq 9;\ x_1, x_2 \geq 0.$$

4. (*a*) If x solves the linear program LP-I and if an optimal solution y to the dual program OLP satisfies $(I - M^T)y + p > 0$ where I is the identify matrix, then show that x solves the corresponding linear complementarity problem.

(*b*) If θ is any strictly increasing function from R to R with $\theta(0) = 0$. Then show that x solves the complementarity problem if and only if

$$\theta(|F_i(x) - x_i|) - \theta(F_i(x) - \theta(x_i) = 0$$

if $i = 1, 2..., n$; where $F_i(x)$ and x_i are the ith components of the vectors $f(x)$ and x respectively.

Or

(*c*) Find an $x \in R^4$ that solves the linear complementarity problem where

$$M = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 2 \\ -1 & -1 & -2 & 0 \end{pmatrix},\ q = \begin{pmatrix} -4 \\ -3 \\ -2 \\ 3 \end{pmatrix}$$

5. (*a*) Using Frank and Wolfe algorithm solve the following program :

Minimize $f(x) = x_1^2 + 4x_2^2$

subject to

$$x_1 + 2x_2 - x_3 = 1;\ -x_1 + x_2 + x_4 = 0;$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

Or

(*b*) Solve the following problem by Kelley's cutting plane algorithm :

Minimize $4x_1 + 5x_2$

subject to

$$x_1^2 + 2x_1x_2 + 2x_2^2 \leq 4;\ -x_1^2 - x_2^2 + 4x_1 \geq 3.$$