MA/MSc-Math.-IVS-(401)

2018

Full Marks : **80**

Time : **3** hours

The figures in the right hand margin indicate marks.
Answer from both the Section as directed.

(GRAPH THEORY)

Section – A

1. *Answer any **four** of the following:* ***(04x04=16)***

(a) Explain fusion of two vertices in a graph with an example.

(b) Draw a self complementary graph and justify it.

(c) If T is a tree with n vertices then show that it has n-1 edges.

(d) Show that a graph with $n(\geq 2)$ vertices has at least two vertices which are not cut vertices.

(e) Show that every tree has either one or two centers.

(f) Prove that the polyhedral graphs of the five platonic solids are Hamiltonian.

OR

2. *Answer **all** questions :* ***(02x08=16)***

(a) What do you mean by the degree of a vertex ?

(b) Define a regular graph with an example.

(c) State Dirac condition for the existence of a Hamiltonian circuit in a simple graph.

(d) Define a center of a tree.

(e) Define a separable graph with an example.

(f) State Kuratowspi's theorem for planes graphs.

(g) Define adjacency matrix of a graph.

(h) State Euler's formula for planer graph.

Section – B

*Answer **all** questions* ***(16x4=64)***

3. (a) Let G be a nonempty graph with at least two vertices. Then show that G is bipartite if and only if it has no odd cycle.

(Turn over)

OR

(b) Prove that a simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges.

4. (a) Let G be a graph with n vertices then show that the following three statement are equivalent

(i) G is a tree

(ii) G is an cyclic graph with *n-1* edges

(iii) G is a connected graph with *n-1* edges.

OR

(b) (i) Show that a graph G is connected if and only if it has a spanning tree.

(ii) Let G be a graph with n vertices and e edges and let $w(G)$ denote the number of connected components of G. Then prove that G has at least $n\text{-}w(G)$ edges.

5. (a) Prove that a connected graph G is Euler if and only if the degree of every vertex is even.

OR

(b) If G is a simple graph with n vertices, where $n \geq 3$ and the degree $d(v) \geq {}^{n}/_{2}$ for every vertex $v \in G$, then prove that G is Hamiltonian.

6. (a) (i) Draw a tree in which its diameter is not equal to twice the radius. Justify it. Under what condition does the equality holds ?

(ii) Prove that the maximum vertex connectivity of a graph with a vertices and e edges $(e \geq n-1)$ is the integral part of the number ${}^{2e}/_{n}$.

OR

(b) Let G be a simple and connected graph. If G contains on sub-graph which is a subdivision of k_5 or $k_{3,3}$, then prove that G is planar.