Or

(*c*) Prove that the linear transformation T on V is unitary off it takes an orthonormal basis of V into an orthonormal basis of V. 8

(*d*) Prove that the Hermitian linear transformation T is non-negative if and only if all of its characteristics roots are non-negative. 8

2018

Time : 3 hours

Full Marks : 80

Answer from both the Sections as per direction

The figures in the right-hand margin indicate marks

Candidates are requlred to answer in their own words as far as practicable

(ALGEBRA-II)

SECTION—A

1. Answer *all* questions : 2 × 8

   (*a*) Define annihilator of a subspace.

   (*b*) Define solvable of a group.

   (*c*) Define idempotent of a linear transformation.

   (*d*) Define eigenvalues of T.

   (*e*) Define invariant under $T \in A(V)$.

(*f*) Define Jordan form of a matrix.

(*g*) Define Jacobson Lemma.

(*h*) State Cramer's Rule.

Or

2. Answer *all* questions : 4 × 4

(*a*) Show that the fixed field of G is a subfield of K.

(*b*) For the given matrix

$$A = \begin{pmatrix} 0 & 0 & 6 \\ 0 & 1 & 1 \\ 6 & 3 & 2 \end{pmatrix} \in F_3$$

find $A^2 - 11A + 3$.

(*c*) Show that if $T \in A(V)$ is nilpotent, then

$\alpha_0 + \alpha_1 T + ... + \alpha_n T^n$,

where the $\alpha_1 \in F$, is invertable if $\alpha_0 \neq 0$

(*d*) Show that if N is normal and $AN = NA$. then $AN^* = N^*A$.

SECTION– B

Answer all questions :

3. (*a*) If V is finite demensional and W is a sub space of V, then show that $\hat{W}$ is isomorphic to $\hat{V}/A(W)$ and dim $A(W)$ = dim V – dim W. 8

(*b*) Let V be a finite dimensional inner product space then show that V has an orthonormal set as a basis. 8

Or

(*c*) Prove that if $P(x) \in F[x]$ is solvable by radicals over F, then the Galo's group over F of $p(x)$ is also solvable group. 8

(*d*) If K is a finite extension of F, then $G(K, F)$ is a finite group and its order, $O(G(K, F)$ satisfies $O(G(K, F)) < [K : F]$. 8

4. (*a*) Show that if V is finite dimensional over F, then $T \in A(V)$ is invertiable iff the constant term of the minimal polynomial for T is not O. 8

(*b*) If $T \in A(V)$ and if $p(x)$ is the minimal polynomial for T over F, suoppose that $p(x)$ has all its roots in F. Prove that every root of $p(x)$ is a characteristic root of T. 8

Or

(*c*) If V is an orbitary vector space over F and if $T \in A(V)$ is right invertiable with unique right inverse, then prove that T is invertiable. 8

(*d*) Prove that if V is n-dimensional over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis $v_1, \ldots, v_n$ and the matrix $m_2(T)$ in the basis $w_1, \ldots, w_n$ of V over F, then there is an element $C \in F_n$ such that

$$m_2(T) = Cm_1(T)C^{-1}.$$ 8

5. (*a*) If $T \in A(V)$ has all its characteristics roots in F, then show that there is a basis of V in which the matrix of T is Triangular. 8

(*b*) Prove that there exists a subspace W of V, invariant under T such that $V = V_1 \oplus W$. 8

Or

(*c*) Prove that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ iff they have the same elementary disvisors. 8

(*d*) Prove that two nilpotent linear transformation are similar if and only if they have the same invariants. 8

6. (*a*) If F is a field of characteristics O, and if $T \in A_F(V)$ is such that $t_r T^i = O$ for all $i > 1$ then T is nilpotent. 8

(*b*) If $A, B \in F_n$ then show that

$$\det(AB) = (\det A)(\det B).$$ 8