Total Pages—7 **MA/M.Sc.—Math-IIS (CC 202)**

2018

Time : 3 hours

Full Marks : 80

Answer from **both** the section as per direction

The figures in the right-hand margin indicate marks

Candidates are required to answer in their own words as far as practicable

Symbols used have their usual meaning

(ADVANCED CALCULUS)

SECTION – A

1. Answer *all* questions from the following : 2 × 8

   (*a*) Define directional derivative of a real valued function at a point.

   (*b*) Define Taylor polynomial of degree *n* at a point x_0.

   (*c*) Define affine transformation.

(Turn Over)

(*d*) What is an implicit function ?

(*e*) Define differentiability of a transformation.

(*f*) State stokes theorem.

(*g*) Define exactness of a 1-form w.

(*h*) What do you mean by functionally dependent of a set of functions ?

Or

2. Answer any *four* of the following : 4 × 4

(*a*) If $f(x, y) = x^2 + 3xy$, find directional derivative of f at (2, 0) in the direction

$$\beta = \left(\frac{1}{2}, -\frac{1}{\sqrt{2}}\right)$$

(*b*) Show that, if $u = \phi(x, y)$, $v = \psi(x, y)$, $x = f(r, \theta)$, $y = g(r, \theta)$,

$$\frac{\partial(u,v)}{\partial(x,y)}\frac{\partial(x,y)}{\partial(r,\theta)} = \frac{\partial(u,v)}{\partial(r,\theta)}.$$

(*c*) Let $f \in c'$ is an open set s suppose f has a

local maximum at a point $p_0 \in S$. Then show that $Df(p_0) = 0$.

(*d*) If γ_1, and γ_2 are smoothly equivalent curves, then show that $L(\gamma_1) = L(\gamma_2)$.

(*e*) Find the point at which the solution of $x^2 = yu = 0$ and $xy + uv = 0$, for u and v can be obtained.

(*f*) If $u = f(x, y)$ and $v = g(x, y)$, then show that

$$dudv = \frac{\partial(u,v)}{\partial(x,y)} dxdy$$

SECTION – B

Answer all questions : 16 × 4

3. (*a*) If $f \in c'$ in an open set s, then show that all its directional derivatives exist at any point $p \in s$ and $D_\beta f(p) = \beta . Df(p)$.

(*b*) If $f \in c'$ in an open convex set S in n-space. then show that for any points p_1 and p_2 in S, their is a point p^* lying on the segment joining them such that

$$f(p_2)-f(p_1)=Df(p^*)\cdot(p_2-p_1)$$

Or

(*c*) Find the polar form of the Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

(*d*) Let $f \in c^{n+1}$ on an interval I about $x=c$, and let $P_c(x)$ be the Taylor polynomial of degree n at c. Then show that $f(x)=P_c(x)+R_n(x)$ for any $x \in I$, where the remainder R_n is given by

$$R_n(x)=\frac{1}{n!}\int_c^x f^{(n+1)}(t)(t-c)^n\,dt$$

4. (*a*) Let T be a transformation of class c' on an open set D in n-space, and let E be a compact set in D. Then show that there are numbers

M and $\delta>0$ such that $|T(p)-T(q)|\le M|p-q|$ for all p, q in E with $|p-q|<\delta$.

(*b*) Let T be a transformation from R^n into R^m which is of class c' in an open set D and suppose that $J(p)=0$ for each $p \in D$. Then show that T is locally are-to-one in D.

Or

(*c*) Show that for any $p \in s$ and $u \in R^3$

$$dg|_p(u)=Dg(p).u$$

(*d*) Let T be of class c' on an open set D in n-space taking values in n-space. Suppose that $J(p)\neq 0$ for all $p \in D$. Then show that $T(D)$ is an open set.

5. (*a*) Let F and G be of class c' in an open set $D \subset R^5$. Let $p_0=(x_0, y_0, z_0, u_0, v_0)$ be a point of D at which both of the equations $F(x, y, z, u, v)=0$, $G(x, y, z, u, v)=0$ are satisfied. Suppose that $\frac{\partial(F,G)}{\partial(u,v)}\neq 0$ at p_0. Then show

that there are two function ϕ and ψ of class c in a neighborhood N of (x_0, y_0, z_0) such that $u = \phi(x, y, z)$, $v = \psi(x, y, z)$ is a solution of the given equations on N giving u_0 and v_0 at (x_0, y_0, z_0).

(*b*) If D^* is a parallelogram bounded by the lines $y = \frac{1}{2}x, y = \frac{1}{2}x + 2, y = 3x, y = 3x - 4$, make an appropriate sobstitution so as to evaluate $\iint_{D^*} xy\,dx\,dy$.

Or

(*c*) Find the length of the curve $y^2 = 4ax$ cut off by its latus rectum.

(*d*) Let T be a transformation from R^3 in to R^3 described by $u = f(x, y, z)$, $v = g(x, y, z)$, $w = h(x, y, z)$ which is of class c' in an open set D, and suppose that at each point $p \in D$ the differential dT has rank 2. Then show that T maps D onto a surcace in UVW space.

(*a*) State and Prove Greens theorem.

(*b*) If α is a k form and β any differentiate form, then show that

$$d(\alpha\beta) = (d\alpha)\beta + (-1)^k \alpha d(\beta).$$

Or

(*c*) Let Σ be a smooth surface of class c' whose domain D is a standard region or a finite union of standard regions in UV plane. Let

$$w = A\,dx + B\,dy + C\,dz$$

where A, B and C are of class c' on i. Then show that

$$\int_{\partial\Sigma} A\,dx + B\,dy + C\,dz = \iint_{\Sigma}\left\{\left(\frac{\partial c}{\partial y} - \frac{\partial A}{\partial z}\right)dy\,dz + \left(\frac{\partial A}{\partial z} - \frac{\partial C}{\partial x}\right)dz\,dx + \left(\frac{\partial B}{\partial x} - \frac{\partial A}{\partial y}\right)dx\,dy\right\}.$$

(*d*) If $w = xy\,dx - y^2 dy$ and γ is given by a square with two apposite vertices (0, 0) and (1, 1) find $\int_\gamma w$.