(*d*) Let f be an integrable function on $[a, b]$ and suppose that

$$F(x) = F(a) + \int_a^x f(t)dt.$$

Then show that $F'(x) = f(x)$ a.e in $[a, b]$.

6. (*a*) State and prove Minkowski inequality.

(*b*) Show that a normal linear space X is complete if and only if every absolutely summable sequence is summable.

Or

(*c*) State and prove Riesz representation theorem.

MA/M.Sc.--Math-IIS(201)

2018

Time : 3 hours

Full Marks : 80

Answer from **both** the Sections as per direction

The figures in the right-hand margin indicate marks

Candidates are required to answer in their own words as far as practicable

Symbols used have their usual meaning

(ABSTRACT MEASURE)

SECTION – A

1. Answer *all* of the following : 2×8

(*a*) Define outer measure of a set.

(*b*) What is almost everywhere property ?

(*c*) Define a simple function.

(*d*) State Fatou's lemma.

(*e*) Define a function of bounded variation.

(*f*) Define a convex function.

(*g*) Define bounded linear functional on a normed linear space.

(*h*) Define L^p space.

Or

2. Answer any *four* of the following :

(*a*) Let $1 \le p < \infty$. Then show that for a, b, t non-negative

$$(a+tb)^p \ge a^p + p^t b\, a^{p-1}$$

(*b*) Show that the function $f(x)$ defined by

$$f(x) = \begin{cases} x \sin\frac{1}{x}, & x \ne 0 \\ 0 \quad , & x = 0 \end{cases}$$

is not a function of bounded variation.

(*c*) Show that

(*i*) $\chi_{A \cap B} = \chi_A \cdot \chi_B$

(*ii*) $\chi_{\bar{A}} = 1 - \chi_A$

(*d*) Show that union of two measurable set is measurable.

(*e*) Show that function defined by

$$f(x) = \begin{cases} 0, & x \text{ irrational} \\ 1, & x \text{ rational} \end{cases}$$

is not integrable in the sense of Riemann.

(*f*) If f is a measurable function and $f = g$ a.e, then show that g is also measurable.

SECTION – B

Answer all questions : 16×4

3. (*a*) (*i*) If $m^*E = 0$, then show that E is measurable.

(*ii*) Show that m^* is translation invariant.

(*b*) Let c be a constant and f and g be two measurable real valued functions defined on the some domain. Then show that

(*i*) cf is measurable

(*ii*) fg is measurable.

Or

(*c*) Let E be a measurable set of finite measure and $\langle f_n \rangle$ a sequence of measurable functions defined on E. Let f be a real valued function such that for each $x \in E, f_n(x) \to f(x)$. Then show that for every $\in > 0$ and $\delta > 0$, there is a measurable set $A \subset E$ with $mA < \delta$ and integer N such that for all $x \notin A$ and $n \geq N$, $|f_n(x) - f(x)| < 0$.

(*d*) Show that the interval (a, ∞) is measurable.

4. (*a*) State and prove bounded convergence theorem.

(*b*) Define convergence is measure of a sequence of measurable function. Let $\langle f_n \rangle$ be a sequence of measurable functions that converges in measure to f. Then show that there is a sub-sequence $\langle f_{n_k} \rangle$ that converges to f a.e.

Or

(*c*) State and prove Lebesgue convergence theorem.

(*d*) Let ϕ and ψ be simple functions which vanish outside a set of finite measure. Then show that for any two scalars a and b.

$$\int a\phi + b\psi = a\int \phi + b\int \psi$$

Further, if $\phi \geq \psi$ a.e, then $\int \phi \geq \int \psi$.

5. (*a*) Let $f(t)$ be integrable on $[a, b]$ and $\int_a^x f(t)\, dt = 0$ for all $x \in [a, b]$, then show that $f(t) = 0$ a.e. in $[a, b]$.

(*b*) Define absolutely continuous function and show that if f is absolutely continuous on $[a, b]$, then it is of bounded variation on $[a, b]$.

Or

(*c*) Show that a function f is of bounded variation on $[a, b]$ if and only if f is the difference of two monotone real-valued fucntions on $[a, b]$.