Registration No. : | | | | | | | | | | |

No. of Pages : 2

B.Tech
BS 1101

First Semester Examinations, 2012-13
MATHEMATICS - I
Full Marks – 70
Time : 3 Hour

Answer question No. 1 which is compulsory and any five from the rest.
The figures in the right-hand margin indicate marks

1. Answer the following questions 2x10
   (a) What is the degree and order of the differential equation whose general solution is $\ln y = Ax + Bx^2$, where A and B are two arbitrary constants? .
   (b) Define a linear differential equation of first order and give one example.
   (c) What do you mean by exact differential equation? How can you know that a differential equation is exact or not?
   (d) Find the general solution of the differential equation $xy^{11} + 2y^1 = 0$
   (e) Define radius of curvature. What is the radius curvature of $(x - 2)^2 + (y + 2)^2 = 16$ at any point .
   (f) Find the asymptote parallel to the co-ordinate axes of the curve $xy^2 + x^2 y + 2xy - y + x + 2 = 0$.
   (g) Explain the idea of power series method to solve a differential equation. .
   (h) Find the radius of convergence of the series $\sum_{m=0}^{\infty} \frac{(-1)^m}{k^m} x^{2m}$.
   (i) What do you mean by linearly independent vectors? Are the following vectors linearly independent?
   $[2 \quad -3]$, $[3 \quad 6]$, $[-1 \quad 4]$.
   (j) Define eigen value and eigen vector.

2. Solve the following differential equations
   (a) $y\,dx + [y + \tan(x + y)]\,dy = 0$. 5
   (b) $(3xe^y + 2y)\,dx + (x^2e^y + x)\,dy = 0$ 5

3. Find the general solution of the differential equations :
   (a) $y^{11} - y = 2e^x$ where, $y(0) = -1$, $y^1(0) = 0$ 5

(b) Use the method of variation of parameter to solve $y^{11} + 9y = \sec 3x$ 5

4. (a) Find a power series solution of the differential equation $(1 - x^2)y^1 = 2xy$ 5

(b) Prove that $\int J_{\nu+1}(x)dx = \int J_{\nu-1}(x)dx - 2J_{\nu}(x)$

Where $J_n(x)$ is the Bessel's function of order n. 5

5. (a) Define the rank of a matrix and find the rank of the following matrix 5

$$\begin{bmatrix} 2 & 0 & 1 & 3 \\ -2 & 4 & 6 & -3 \\ 1 & -4 & 1 & -5 \end{bmatrix}.$$

(b) Solve the following linear system of equations by Gauss elimination method 5

$$x_1 - 2x_2 + 3x_3 = 0$$
$$-2x_1 + x_2 - 4x_3 = 3$$
$$10x_2 + 5x_3 = 9$$
$$6x_1 + 10x_2 = 8 .$$

. 5

6. (a) Find the spectrum and eigenvectors of the matrix 5

$$\begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}.$$

(b) Find out what type of conic section is represented by the following quadratic form and transform it to principal axes.

$41x_1^2 - 24x_1x_2 + 34x_2^2 = 156$ 5

7. (a) Find a basis of eigenvectors and diagonalize the following matrix . 5

$$\begin{bmatrix} 16 & 0 & 0 \\ 48 & -8 & 0 \\ 84 & -24 & 4 \end{bmatrix}.$$

(b) Show that the radius curvature at a point of the curve

$x = ae^{\theta}(\sin\theta - \cos\theta)$, $y = ae^{\theta}(\sin\theta + \cos\theta)$

is twice the distance of the tangent at the point from the origin.

5

8. (a) Find the asymptote of the curve $2x(y - 3)^2 = 3y(x - 1)^2$ 5

(b)If a right line is drawn through the point (a ,0) parallel to the asymptote of the cubic $(x - a)^3 - x^2y = 0$, prove that the portion of the line intercepted by the axes is bisected by the curve. 5