Registration No : | | | | | | | | | | |

Total Number of Pages : 03 **B.Tech.**

PCE4E001

4th Semester Regular / Back Examination 2017-18

APPLIED MATHEMATICS – III

BRANCH : CHEM

Time : 3 Hours

Max Marks : 100

Q.CODE : C585

Answer Part-A which is compulsory and any four from Part-B.

The figures in the right hand margin indicate marks.

Answer all parts of a question at a place.

<u>Part – A (Answer all the questions)</u>

Q1 **Answer the following questions:** ***multiple type or dash fill up type:*** **(2 x 10)**

a) Let f(z) and g(z) has pole of order '20' and '18'at z = 1 respectively then what is the order of pole of the function f(z)×g(z) at z = 1

(a) 36 (b) 38 (c) 2 (d) –2

b) If f(z) has removal singularity at $z = \infty$,then f(z) is equal to

(a) $\sin z$ (b) $\cos z$ (c) $\tan z$ (d) none

c) If both real part and imaginary part of a complex function are harmonic conjugate to each other then the function must be

(a) Analytic (b) Differentiable (c) constant (d) none

d) Let $f(z) = \tan(\frac{1}{z}), then\ z = 0$ is

(a) Removal singular point (b) Isolated essential singular point

(c) pole (d) non isolated essential singular point

e) Simpson's rule is exact for a polynomial of degree

(a) 5 (b) ≤ 5 (c) 2 (d) ≤ 2

f) Let $\frac{dy}{dx} = -2xy^2, y(0) = 1$ with step size h = 0.2 then by Euler method the value of $y(0.4)$ is _______________.

g) Using Lagrange interpolation how many nodes or arguments are required to obtain a polynomial of degree 10

(a) 11 (b) 12 (c) 21 (d) 10

h) Two dice are rolled, the probability of even doublet occur on the surface is _____________?

i) Let the cumulative distribution function of the random variable X is given by

$$F(x) = \begin{cases} 0 \ ; & x < 0 \\ x; & 0 \leq x < \frac{1}{2} \\ \frac{1+x}{2}; & \frac{1}{2} \leq x < 1 \\ 1; & x \geq 1 \end{cases}$$

Then P(X= $\frac{1}{2}$) = ______________?

j) The distribution function of random variable X is given by

$$F(x) = \begin{cases} cx^3 \ ; & 0 \leq x < 3 \\ 1 \ ; & x \geq 3 \\ 0; & x < 0 \end{cases}$$

Then c = __________?

Q2 **Answer the following questions: *Short answer type:*** **(2 x 10)**

a) Find the Radius of convergence of $\sum_{n=0}^{\infty} a_n z^n$ where

$$a_n = \begin{cases} \left(\dfrac{1+i\sqrt{3}}{2}\right)^n, & n \text{ is even} \\ \left(\dfrac{1-i}{3}\right)^n, & n \text{ is odd} \end{cases}$$

b) Evaluate $\int_{c:|z|=3} \frac{e^{2z}dz}{(z+1)^4}$

c) Find the principal value of $(-1)^{\frac{-5i}{\pi}}$.

d) Residue of f$(z) = \frac{\sin z - z}{z^3}$ $at\ z = 0$ is _____________?

e) What is the approximate value of $\int_0^1 \frac{x}{x+1}dx$ by Simpson's rule?

f) If p$(x) = 2 + \alpha(x+1) + x(x-\beta)$ interpolates the points (x,y) in the table

x	-1	0	1
y	2	1	2

Then $+\beta =$ _____________ ?

g) Find the unique polynomial of degree ≤ 2 such that $f(0) = 1, f(1) = 3$ and $f(3) = 55$.

h) Fit a straight line using the following data y = a + bx

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

i) The Random variable X can takes on two values 2 and 4, where P(X=2) = 0.2and P(X=4) = 0.8 then find E(X^2) .

j) What is the probability mass function of poisson distribution?

Part – B (Answer any four questions)

Q3 **a)** Evaluate $\oint_{|z|=4} \frac{\cos z\, dz}{z^2(z-\pi)^2}$ by Residue theorem. **(10)**

b) Find the analytic function f(z)= u + iv whose real part u = $e^{-2x}\sin 2y$. **(5)**

Q4 **a)** Show that $\int_{-\infty}^{\infty} \frac{dx}{1+x^4} = \frac{\pi}{\sqrt{2}}$ by contour method. **(10)**

b) Evaluate the contour integration $\int_0^{2\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta$ **(5)**

Q5 **a)** Find the interpolating polynomial and an approximation to the value of f(10) using the following data points **(10)**

x	0.5	1.5	3.0	5.0	6.5	8.0
f(x)	1.625	5.875	31.000	131.000	282.125	521.0

b) Find $\tan(0.12)$ using the following data points **(5)**

x	0.10	0.15	0.20	0.25	0.30
y=tan x	0.1003	0.1511	0.2027	0.2553	0.3093

Q6 a) Let $\frac{dy}{dx} = y - x^2 + 1, y(0) = 0.5$ then solve it **(10)**
using modified Euler method to find $y(0.2)$ with step size h = 0.2
using Runge-Kutta fourth order method find $y(0.4)$ with h = 0.4

b) Evaluate $\int_0^2 \frac{dx}{x^2+4}$ using Simpson's rule and Romberg's method for step size h = 0.5 **(5)**

Q7 a) Expand $f(z) = \frac{1}{z^2-3z+2}$ in the following regions **(10)**

(a) 0<$|z| < 1$ (b)1<$|z| < 2$ (c) $|z| > 2$

b) Let R be the radius of convergence of $\sum_{n=0}^{\infty} a_n z^n$ then find the radius of convergence of (a) $\sum_{n=0}^{\infty} (a_n)^2 z^n$ **(5)**

Q8 a) If the mathematics scores of BPUT examination are normal with mean 70 and standard deviation 20 and if some companies sets 75 as the minimum score for qualifying to sit in campus drive, then what percent of students will not reach that score. **(10)**

b) A continuous Random variable X, has the probability density function **(5)**

$$f(x) = \begin{cases} \frac{2x}{R^2}; & 0 < x < R \\ 0; & \text{Otherwise} \end{cases}$$ then find the Mean and Variance of the distribution.

Q9 a) Find the equations of two lines of regression for the following data .Also obtain the estimation of X for Y= 54 **(10)**

X	27	25	22	20	18	17	16
Y	70	69	68	62	56	54	50

b) A normally distributed random sample of size 100 observations is taken from its population with standard deviation 8 and unknown mean μ.A statistician uses the critical region $\bar{x} > 43$ for testing the null Hypothesis
H_0:$\mu = 41$.Calculate power of the test w.r.t. alternate hypothesis
H: $\mu = 42$ **(5)**