Registration No : | | | | | | | | | | |

Total Number of Pages : 02 **B.Tech.**

BSCM1211

4th Semester Back Examination 2017-18
DISCRETE MATHEMATICS
BRANCH : CSE, IT, ITE
Time : 3 Hours
Max Marks : 70
Q.CODE : C584

Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.
Answer all parts of a question at a place.

Q1 **Answer the following questions:** **(2 x 10)**

a) Translate into a logical expression: "You can access the internet from campus only if you are a computer science major or you are not a freshman"

b) Construct the truth table for $(p \wedge q) \rightarrow (p \wedge q)$.

c) Write the principal disjunctive normal form of $(p \wedge \neg q)$

d) Express the statement "Every student in this class has studied calculus" as a universal quantification.

e) Prove that a tree with n vertices has $n-1$ edges.

f) Give an example of graph having Hamiltonian circuit but not an Eulerian circuit.

g) Prove that in an undirected graph numbers of odd degree vertices are even.

h) Define Monoid and semigroup.

i) In Boolean Algebra if $a+b=1$ & $a.b=0$,show that the complement of every a element is unique.

j) Simplify the Boolean expressions $XY + X'Z + YZ$

Q2 **a)** Show that the propositions are $p \vee (q \wedge r)$ & $(p \vee q) \wedge (p \vee r)$ logically equivalent. **(5)**

b) Using rule of inference, determine whether the conclusion C is valid in the following premises : **(5)**

$$H_1 : P \Rightarrow (Q \Rightarrow R)$$
$$H_2 : P \wedge Q$$
$$C : R$$

Q3 **a)** Prove that for any integer $n > 1$ $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ...+ \frac{1}{\sqrt{n}} \geq \sqrt{n}$. **(5)**

b) Solve the recurrence relation $a_n = 4(a_{n-1} - a_{n-2})$ with initial condition $a_0 = a_1 = 1$. **(5)**

Q4 Let $R = \{(1,2), (2,3), (3,1)\}$ and $A = \{1,2,3\}$,find the reflexive, symmetric and transitive closure of R ,using **(10)**

(i) Composition of relation R .

(ii) Composition of matrix relation R

Q5 **a)** Let G be a connected planar simple graph with E is number of edges & V is the number of vertices & R is a number of regions then $V - E + R = 2$. **(5)**

b) State & prove five-color theorem for groups. **(5)**

Q6 **a)** Prove that the number of edges in a bipartite graph with n vertices is at most $\left(\frac{n^2}{2}\right)$. **(5)**

b) Let $(L, \leq)$ be a lattice. Thenfor $a, b, c, d \in L$. **(5)**

(i) $a \leq b \Rightarrow a \vee c \leq b \vee c$.

(ii) $a \leq b \Rightarrow a \wedge c \leq b \wedge c$.

Q7 **a)** Show that every finite lattice has a least upper bound & a greatest lower bound. **(5)**

b) Show that if (a, b) are arbitrary elements of a group G, then $(ab)^2 = a^2 b^2$ if G is abelian. **(5)**

Q8 **Answer any TWO :** **(5 x 2)**

a) Prim's Algorithm

b) Kruskal's Algorithm

c) Eulerian and Hamiltonian Graph

d) Principal Ideal Domain& Maximal Ideal