Registration No :

Total Number of Pages : 02 **B.Tech.**

PAT2A001

2nd Semester Regular / Back Examination 2017-18
APPLIED MATHEMATICS - II
BRANCH : AEIE, AERO, AUTO, BIOMED, BIOTECH, CHEM, CIVIL, CSE, ECE, EEE, EIE, ELECTRICAL, ENV, ETC, FAT, IEE, IT, MANUFAC, MANUTECH, MECH, METTA, MINERAL, MINING, MME, PE, PLASTIC, PT, TEXTILE
Time : 3 Hours
Max Marks : 100
Q.CODE : C602
Answer Part-A which is compulsory and any four from Part-B.
The figures in the right hand margin indicate marks.
Answer all parts of a question at a place.

Part – A (Answer all the questions)

Q1 Answer the following questions: ***multiple type or dash fill up type:*** **(2 x 10)**

a) $L^{-1}[\frac{s+3}{S^2-s-2}]$ =_______?

b) The Laplace transformation $of the function\, f(t) = (5^t)$ is _______?

c) The fundamental period of f(x)= $10^{100}sin^2x + 10^{100}cos^2x$ is _______?

d) Using Gamma function finds the value of $\int_0^\infty x^6 e^{-2x}dx$ is __________?

e) curl (grad f) = ___________?

f) The Fourier sine transformation of the function f(x)= e^{-ax}(a>0) is _______?

g) The value of $\int_C F(r)\cdot dr, where\, F = [y^2, -x^2]$ and C: Be the line segment from (0, 0) to (1, 4) is _________?

h) The value of integral $\int_0^1 x^4(1-x)^2 dx$ is ______

i) The value of t $* sint$ is _________?

j) The value of the constant 'b' such that f$(x, y, z) = [bx^2y + yz,\ xy^2 - xz^2,\ 2xyz - 2x^2y^2]$ has divergence zero is ____________?

Q2 Answer the following questions: ***Short answer type:*** **(2 x 10)**

a) Find the Laplace transformation of the function f$(t) = cosh\, at\, sinh\, bt$

b) Find $\nabla^2 f \;\; where \;\; f = e^{2x}sin2y$.

c) Write the sufficient condition for existence of Laplace transformation of a function.

d) Find the Directional derivative of the function f = $x^2 + y^2$ at a point p $(1,1)$ in the direction $\vec{a} = 2\hat{\imath} - 4\hat{\jmath}$

e) State Green's theorem in plane.

f) Find the Laplace transformation of the unit impulse function $\delta(t - 2^{2017})\ and$ The unit step function $U(t - 2^{2017})$

g) Find the Fourier sine series of the function f(x)= $-100^{10}(-\pi < x < \pi)$; f(x)= $100^{10}(0 < x < \pi)$

h) Find a parametric representation of the Parabolic equation $z = 9(x^2 + y^2)$

i) Find L[f(t)], Where $f(t) = \begin{cases} 1; 0 < t < 1 \\ 2; 2 < t < 4 \\ 0; \quad\quad t > 4 \end{cases}$

j) Find the value of $L^{-1}\left[\frac{s^2+6}{(s^2+1)(s+4)}\right]$

Part – B (Answer any four questions)

Q3 **a)** Solve the following initial value problem using Laplace transformation **(10)**

$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = \cos 2t \ with \ y(0) = 2, y'(0) = 1$

b) Show that L$\left(\frac{\cos\sqrt{t}}{\sqrt{t}}\right) = \sqrt{\frac{\pi}{s}}e^{(-1/4s)}$ **(5)**

Q4 **a)** Verify Green's Theorem in the plane for $\oint_C (xy+y^2)dx + x^2dy$, where 'C' is the closed curve of the region bounded by y= $x^2 \ and \ y = x$ **(10)**

b) Find the area bounded by one arch of the cycloid x= $a(\theta - \sin\theta), y = a(1 - \cos\theta)$; 0≤ $\theta \leq 2\pi$ **(5)**

Q5 **a)** Prove that the integral $\int_0^\infty \frac{\sin\omega\cos\omega x}{\omega}d\omega = \begin{cases} \frac{\pi}{2}\ ; 0 \leq x < 1 \\ \frac{\pi}{4}\ ; \quad x = 1 \\ 0\ ; \quad x > 1 \end{cases}$ **(10)**

b) Prove that $\Gamma(-\frac{7}{2}) = \frac{2^4\sqrt{\pi}}{105}$ **(5)**

Q6 **a)** Solve the following integral equation using Laplace transformation **(10)**

$y(t) = 1 + \int_0^t \cos(t-u)y(u)du$

b) Using convolution prove that **(5)**

$2 * 2 * 2 * \cdots\cdots * 2(upto\ 'K'\ times) = \frac{2^K t^{K-1}}{(K-1)!}$

Q7 **a)** Verify Stokes Theorem, when F= $y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$ and surface 'S' Is the part of the sphere $x^2 + y^2 + z^2 = a^2$ above the x-y plane. **(10)**

b) Find the total Mass of a mass distribution of density f$(x,y,z) = e^{-x-y-z}$ in a region T: 0≤ $x \leq 1 - y, 0 \leq y \leq 1, 0 \leq z \leq 2$ **(5)**

Q8 **a)** Verify Divergence Theorem for F= $z\hat{i} + x\hat{j} - yz\hat{k}$ taken over the surface of the cylinder $x^2 + y^2 = 9$ included in the first octant between $z = 0 \ and \ z = 4$ **(10)**

b) Find the coordinates of the center of gravity of a mass of density f$(x,y) = 1$ in the region R : the triangle with vertices $(0,0), (b,0) \ and \ \left(\frac{b}{2}, h\right)$ **(5)**

Q9 **a)** Find the Fourier Transformation of $f(x) = \begin{cases} 0\ , & x < 0 \\ e^{-x^2}, & x > 0 \end{cases}$ **(10)**

b) Find the Fourier series expansion of $f(x) = \begin{cases} \frac{1+2x}{2} & if\ -\frac{1}{2} < x < 0 \\ \frac{1-2x}{2} & if \quad 0 < x < \frac{1}{2} \end{cases}$ **(5)**