Registration No. :

Total number of printed pages – 3

First Semester Examination – 2013

MATHEMATICS – I

QUESTION CODE : C- 609

Full Marks – 70

Time : 3 Hours

*Answer Question No. 1 which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions :   2×10

    (a) What is the degree and order of the differential equation whose general solution is

    $\sin y = ax^2 + be^x$, where a and b are two arbitrary constants ?

    (b) Define a linear differential equation of second order and give one example.

    (c) What do you mean by integrating factor ? How it helps to solve a differential equation ?

    (d) Find the general solution of the differential equation $y^{11} - 8y = 0$.

    (e) Find the radius curvature at any point on the curve $y = c \cosh x/c$.

    (f) Find the asymptotes of the curve $y(x - y)^2 = x + y$.

    (g) What does convergence of power series mean ? How would you test it ?

    (h) Find the radius of convergence of the series $\sum_{m=0}^{\infty} \dfrac{x^{2m+1}}{(2m + 1)!}$.

    (i) What is the rank of a matrix ? Why is it of basic importance ?

    (j) What is a basis of eigen vectors ? When does it exist ?

P.T.O.

2. Solve the following differential equations

   (a) $\dfrac{dy}{dx} - (1 + \dfrac{3}{x})y = x + 2$, $y(1) = e - 1$ 5

   (b) $(2x + e^y)\,dx + xe^y\,dy = 0$ 5

3. Find the general solution of the differential equations :

   (a) $xy^{11} - y^1 = (3 + x)x^2 e^x$ 5

   (b) $y^{11} - 2y^1 + y = e^x / x^3$ 5

4. (a) Find a power series solution of the differential equation   5

        $(1 - x^2)y^{11} - 2xy^1 + 2y = 0$

   (b) Show that $P_n(x) = \dfrac{1}{2^n n!} \dfrac{d^n}{dx^n}[(x^2 - 1)^n]$ 5

5. (a) Solve the following system of linear equations :   5

        $2x + 3y - z = 0$; $5x - 3y + z = 7$; $8x + 9y - 3z = 2$

   (b) Find the symmetric coefficient matrix C of the quadratic form $Q = x^T C x$

        given by

        $4x_1^2 - 8x_1x_2 + 5x_2^2$ 5

6. (a) If $\lambda_1, \lambda_2, ..., \lambda_k$ be distinct eigenvalues of an $n \times n$ matrix, then show

        that the corresponding eigen vectors $x_1, x_2, ..., x_k$ form a linearly

        independent set.   5

   (b) Find out what type of conic section is represented by the following

        quadratic form and transform it to principal axes.   5

        $9x_1^2 - 6x_1x_2 + x_2^2 = 40$

7. (a) Find a basis of eigen vectors and diagonalize the following matrix : 5

$$\begin{bmatrix} 15 & 6 & -12 \\ 4 & 10 & -2 \\ -4 & 8 & -7 \end{bmatrix}$$

(b) Find the radius of curvature of the curve $r = a(1+\cos\theta)$ at the point where the tangent is parallel to the initial line. 5

8. (a) Find the asymptote of the curve 5

$$(x + y)(x - y)(2x - y) - 4x(x - 2y) + 4x = 0$$

(b) Find the equation of the straight line on which lie three points of intersection of the cubic

$$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$$ 5