Registration No. : | | | | | | | | | | |

Total number of printed pages – 3

B. Tech
BS 1101

First Semester Regular Examination – 2014

MATHEMATICS – I

BRANCH : B. TECH

QUESTION CODE : H 451

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) Define Integrating factor.

(b) Under what condition $(Ax + By)dx + (Cx + Dy)dy = 0$ is exact.

(c) Define asymptote of a curve.

(d) Find the radius of convergence of the series $\sum \frac{(2m)!}{(2m+2)(2m+4)} x^{2m}$.

(e) Write the formula for radius of curvature in pedal form

(f) Prove that $(n+1)\, p_{n+1}(x) = (2n+1)x\, p_n(x) - n\, p_{n-1}(x)$

(g) What is the relation between Bessel's function and gamma function ?

(h) Verify whether the vectors (1,2,3,4),(2,3,4,5) and (3,4,5,6) are linear independent or not.

P.T.O.

(i) Define normalized Eigen vector.

(j) Find the symmetric co-efficient 'c' of the quadratic form $Q = x^T c\, x$ given by $(x_1 - x_2 + 4x_3)^2 - 4(x_2 - x_4)^2$.

2. (a) Solve the initial value problem $(e^{x+y} + ye^y)dx + (xe^y - 1)\,dy = 0$, $y(0) = -1$. 5

(b) Solve $x\,dy - \{y + xy^3(1 + \log x)\}dx = 0$ 5

3. (a) Find the general solution of $y'' + 4y' + 5y = 4x \sin x$. 5

(b) Solve $(x^2D^2 + xD - 4I)\,y = x^{3/2}\cos x$. 5

4 (a) Check whether the solutions of $y'' - 2y' + y = 0$ are linearly dependent or linearly independent. 5

(b) Using method of undetermined co-efficient, solve $y'' + 2y' + y = 2x\sin x$. 5

5 (a) Solve the initial value problem $y'' + 5y' + 6y = 108x^2$, $y(0) = 18$, $y'(0) = -26$ 5

(b) Using variation of parameter solve $(D^2 - 2D + 25)y = e^x \operatorname{cosec} x$. 5

6 (a) Find asymptotes of the following curve $(a+x)^2(b^2 + x^2) = x^2y^2$. 5

(b) Find a power series solution of $y'' - y = 0$ in powers of $x - 1$. 5

7 (a) Solve the differential equation reducing it to Bessel,s function 5

$x^2y'' + xy' + 4(x^4 - v^2)y = 0$.

(b) Prove that the radius of curvature of an asteroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ is equal to 3 times the length of perpendicular from origin to the tangents. 5

8. (a) Solve the following system of equations using Gauss elimination method : 5

$$13x + 12y + 6 = 0$$

$$-4x + 7y + 73 = 0$$

$$11x - 13y - 157 = 0$$

(b) Find the eigen value and corresponding eigen vectors of the following matrix : 5

$$A = \begin{bmatrix} -10 & 10 & -15 \\ 10 & 5 & -30 \\ -5 & -10 & 0 \end{bmatrix}.$$