Registration No. : | | | | | | | | | | |

Total number of printed pages – 2

B. Tech
BSCM 2101

First Year Special Examination – 2014

MATHEMATICS – I

BRANCH(S) : BIOTECH, CSE, EC, EEE, ETC, IT, TEXTILE

QUESTION CODE : G 351

Full Marks – 70

Time : 3 Hours

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*Answer Question No. 1 which are compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

   (a) What is the curvature of the cycloid $s = \sin\Psi$ at $\Psi = \pi/6$?

   (b) What is the only asymptote to the curve $y = \ln x$?

   (c) Solve the equation $(1-x)dy - (1+y)dx = 0$

   (d) If the two independent solution (bases) of a linear homogeneous ordinary differential equation are $\cos(2\ln x)$ and $\sin(2\ln x)$ then write down the equation.

   (e) If $y_1 = e^{3x}$ and $y_2 = xe^{3x}$ then write down the Wronskian of y_1 and y_2.

   (f) Find the radius of curvature of the power series $\sum_{m=0}^{\infty} \frac{(x-3)^m}{3^m}$

   (g) Obtain the Legendre's polynomial $P_2(x)$ using Rodrigues's formula.

   (h) Find the Laplace transform of $e^{-t}\sin 5t$.

   (i) Find Laplace inverse of $\frac{e^{-4s}}{s^2}$

   (j) Find the convolution $t * t$.

2. (a) Find the radius of curvature of the curve $x = a\cos 3t$, $y = a\sin 3t$ at $t = \pi/4$. 5

(b) Find all the asymptotes of 5

$$x^4y + 2x^3y^2 - x^2y^3 - 2xy^4 - x^3y + xy^3 + x^2 + y^2 + 1 = 0$$

3. (a) Trace the curve $y = x^3$ by proper investigation. 5

(b) Solve the equation $\frac{dy}{dx} + y\cos x = y^n \sin 2x$ 5

4. (a) A thermometer, reading 10°C is brought into a room whose temperature is 23°C. Two minutes later the thermometer reading is 18°C. How long will it take until the reading is 22.8°C ? 5

(b) Solve the equation : $(\cos wx + w\sin wx)dx + e^x dy = 0$, $y(0) = 1$ 5

5. (a) Solve $x^2y'' - 2xy' + 2y = 0$, $y(1) = 1.5$ and $y'(1) = 1$. 5

(b) Solve $y'' - 4y = e^{-2x} - 2x$ 5

6. (a) Find the current I at any time t for the RLC circuit with R = 8 ohms, L = 2 henrys, C = 0.1 farad, E = 10 volts with zero initial current and charge. 5

(b) Prove the Bonnet's recursion formula for Legendre's polynomial 5

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$$

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7. (a) Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$ 5

(b) Solve $y'' + y = r(t)$, $y(0) = y'(0) = 0$, $r(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$ 5

8. (a) Solve the integral equation using Laplace Transform 5

$$y(t) + 2\int_0^t y(\tau)\cos(t-\tau)d\tau = \cos t$$

(b) Solve the given system of differential equations using Laplace Transform 5

$$y_1' = 5y_1 + y_2, \quad y_2' = y_1 + 5y_2, \quad y_1(0) = 1, \; y_2(0) = -3$$