Registration No. : | | | | | | | | | | |

Total number of printed pages – 2

B. Tech
BS 1101

First Year Special Examination – 2014

MATHEMATICS – I

BRANCH(S) : AEIE, AUTO, BIOTECH, CHEM, CIVIL, CSE, EC, EEE, ELECTRICAL, ENV, ETC, FASHION, IEE, IT, MANUFACT, MECH, MM, MME, PLASTIC, TEXTILE

QUESTION CODE : G 354

Full Marks – 70

Time – 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) What is the order and degree of the following differential equation :

$$\left(1+\frac{d^2y}{dx^2}\right)^3 = \frac{dy}{dx}$$

(b) Find whether the vectors (1 1 3), (5 2 4), (3 2 6) are linearly independent or not.

(c) Write the formula for radius of curvature in pedal form.

(d) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^{n-1}}{(n+1)}$.

(e) Define Bessel's function of 2nd kind.

(f) What is the value of $p_{2n+1}(0)$, the Legender polynomial of degree $2n+1$.

(g) Define Eigen value and Eigen vector of a matrix.

(h) Define orthogonal matrix, unitary matrix.

(i) Find the symmetric coefficient matrix of the quadratic form

$$Q = 2x_1^2 + 4x_1x_2 + 4x_2x_3 - 9x_3^2.$$

(j) What is algebraic multiplicity and geometric multiplicity of an eigen value.

P.T.O.

2. (a) Solve the Bernoulli's equation $y' - 2xy = 2xy^2$. 5

(b) Solve the initial value problem $ye^x\,dx + (2y + e^x)\,dy = 0,\ y(0) = -1$. 5

3. (a) Solve : $(D^4 - 3D^2 - 4)y = 5\sin 2x - e^{-2x}$. 5

(b) Solve : $\frac{d^2y}{dx^2} + 9y = \sec 3x$ using variation of parameter. 5

4. (a) Solve : $y'' + 4y = 8x^2$ using undetermined coefficient. 5

(b) Reduce the differential equation $xy'' + 5y' + xy = 0$ to bessel's function. 5

5. (a) Find a power series solution of the differential equation $y'' - 3y' + 2y = 0$. 5

(b) Prove that $\int J_{v-1}(x)\,dx = \int J_{v-1}(x)\,dx - 2J_v(x)$ where $J_n(x)$ is the Bessel's function of order n. 5

6. (a) Find the asymptotes of the cubic curve

$$2x^3 - x^2y + 2xy^3 + y^3 - 4x^3 + 8xy - 4x + 1 = 0.$$ 5

(b) Show that the radius curvature at a point of the curve

$$x = ae^{\theta}(\sin\theta - \cos\theta),\quad y = ae^{\theta}(\sin\theta + \cos\theta)$$

is twice the distance of the tangent at the point from the origin. 5

7. (a) Solve the following system of equations $4y + 3z = 8$

$2x - z = 2$

$3x + 2y = 5$ 5

(b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & -1 & 2 \\ 2 & 4 & 11 \end{bmatrix}$ using Gauss-jordan method. 5

8. (a) Show that the eigen value of a hermitian matrix are real. 5

(b) Diagonalize the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ and hence find $x^{-1}A^2x$. 5