Registration No. : | | | | | | | | | | |

Total number of printed pages – 2

B. Tech

BSCM 2102

First Year Special Examination – 2014

MATHEMATICS – II

BRANCH(S) : BIOTECH, CSE, EC, EEE, ELECTRICAL, IT

QUESTION CODE : G 609

Full Marks – 70

Time – 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) Prove that if A is a non-singular square matrix, then A^{-1} is also nonsingular.

(b) Verify the following matrix is symmetric or skew-symmetric

$$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 1 \end{bmatrix}$$

(c) Show that the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is unitary.

(d) Define Hermitian, Skew-Hermitian, orthogonal, unitary matrices.

(e) Find the projection of $\mathbf{a} = (1, 4, 2)$ over $\mathbf{b} = (3, 2, 1)$.

(f) Define directional derivative of a vector field.

(g) Define exactness of an integral.

(h) State Gauss divergence theorem.

(i) What is a half range series ?

(j) Write the formula for Fourier series expansion of any function of period 2l.

P.T.O.

2. (a) Solve the system of equations using Gauss elimination method 5

$$3x_1 + 2x_2 + 7x_3 = 4$$
$$3x_1 + 4x_2 + x_3 = 7$$
$$2x_1 + 3x_2 + x_3 = 5$$

(b) Find a basis of the eigen vectors and diagonalize the following matrix : 5

$$\begin{bmatrix} -43 & 77 \\ 13 & 93 \end{bmatrix}$$

3. (a) Prove that every square matrix can be expressed as sum of a symmetric and skew symmetric matrix. 5

(b) Find the type of conic section represented by the following quadratic form $7x^2 + 6xy + 7y^2 = 200$. 5

4. (a) Find the angle between two surfaces $x + y + z = 8$ and $2x + y - z = 3$. 5

(b) If $f(x, y, z) = x^2 + y^2 - z$, calculate curl (grad f). 5

5. (a) Find the scalar potential of a vector field is given by 5

$$\mathbf{A} = (x^2 + xy^2)\,\mathbf{i} + (x^2y + y^2)\,\mathbf{j}$$

(b) Find directional derivative of $f = xy^2 - 3xyz$ at (1, 2, 2) in the direction of normal to the surface $x^2 + y^2 - z^2 = 1$ at (1, 1, 1). 5

6. (a) Find the integral $\int f.dr$ where f = (2z, x, −y) , r = (cost, sint, 2t) from (1, 0, 0) to (1, 0, 4π). 5

(b) Find the surface integral $\iint F.ndA$, where F = [x − z, y − x, z − y], S : r = [u cos v, u sin v, u], $0 \le u \le 3$, $0 \le v \le 2\pi$. 5

7. (a) Determine whether the line integral

$\int 2xyz^2dx + \left(x^2z^2 + z\cos yz\right)dy + \left(2x^2yz + y\cos yz\right)dz$ is independent of the path of integration. If so, then evaluate it from (1, 0, 1) to (0, π/2, 1). 5

(b) Verify Stokes theorem for a vector field defined by $F = (x^2 - y^2)I + 2xyj$ in the rectangular region in the xy-plane bounded by the lines x = 0, x = a, y = 0, y = b. 5

8. (a) Find the Fourier series expansion of $f(x) = \pi \sin \pi x$, $0 < x \le 1$. 5

(b) Find the Fourier cosine series expansion of $f(x) = x$, $0 < x < \pi$ and $f(x) = \pi - x$, $\pi < x < 2\pi$. 5
