

Registration No. :

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Total number of printed pages – 3

B. Tech
BS 1104

First Year Special Examination – 2014

MATHEMATICS – II

BRANCH(S) : AEIE, AUTO, BIOTECH, CHEM, CIVIL, CSE,
EC, EEE, ELECTRICAL, ENV, ETC, FASHION, IEE,
IT, MECH, MM, MME, TEXTILE

QUESTION CODE : G 613

Full Marks – 70

Time – 3 Hours

Answer Question No. 1 which is compulsory and any **five** from the rest.
The figures in the right-hand margin indicate marks.

1. Answer the following questions :

2 × 10

- Define Dirac's delta function.
- Find the convolution of $\sin t * \cos t$.
- Write the formula for Fourier transform of derivatives.
- What is Fourier integral representation ?
- If $f = xy - yz$, then find $\text{grad}(f^2)$.
- Define vector field and scalar field.
- What is the geometrical significance of curl ?
- State Laplace's equation and its importance.

(i) Evaluate $\int_0^2 \int_y^{y^2} (x^2y + 3x) dx$

P.T.O.

(j) How can we convert line integrals to surface integrals? State the conditions in the corresponding theorem.

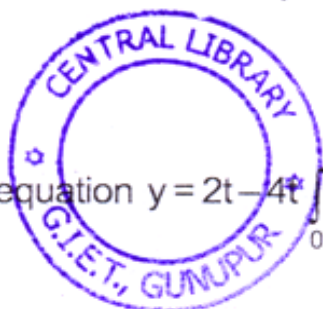
2. (a) Solve the given initial value problem by Laplace transform

$$Y'' - 2y' + 2y = 8 e^{-t} \cos t, \quad y(0) = 16, \quad y'(0) = -16 \quad 5$$

(b) Find the inverse Laplace transform of the given function 5

(i) $\frac{s+1}{s^2} e^{-s}$

(ii) $\frac{3s+4}{s^2+4s+5}$



3. (a) Solve the integral equation $y = 2t - 4t \int_0^t y(\tau)(t - \tau) d\tau$ using Laplace Transform. 5

(b) Find Fourier series expansion of $f(x) = \cos ax$ in the range 0 to 2π . 5

4. (a) Find Fourier transform of $f(x) = e^{-x}$ if $x \geq 0$ and 0 if $x < 0$ 5

(b) Find the Fourier cosine integral representation of 5

$$f(x) = \begin{cases} x^2, & 0 < x < 1 \\ 0 & x > 1 \end{cases}$$

5. (a) Find the directional derivative of $f = 3xyz - xy^2$ at $(1, 2, 2)$ in the direction of normal to the surface $x^2 + y^2 - z^2 = 1$ at $(1, 1, 1)$. 5

(b) Prove that $\text{div}(\mathbf{u} \times \mathbf{v}) = \mathbf{v} \cdot \text{curl} \mathbf{u} - \mathbf{u} \cdot \text{curl} \mathbf{v}$. 5

6. (a) Find a tangent vector and its corresponding unit tangent vector of the curve $\mathbf{r}(t) = \cos t \mathbf{i} + 2\sin t \mathbf{j}$ at the point $(1/2, \sqrt{3}, 0)$ 5

(b) Find the scalar potential of $\mathbf{V} = [ye^x, e^x, 1]$ 5

7. (a) Determine whether the line integral $\int \cos(x + yz)(dx + zdy + ydz)$ is independent of the path or not. If so, then evaluate it from $(1, 0, 1)$ to $(0, \pi/2, 1)$. 5

(b) Find the moment of inertia of a lamina $S : x^2 + y^2 = z^2, 0 \leq z \leq h$ of density 1 about the z-axis. 5

8. (a) Verify Stokes theorem for a vector field defined by $\mathbf{F} = (x^2 - y^2) \mathbf{i} + 2xy \mathbf{j}$ in the rectangular region in the xy-plane bounded by the lines $x=0, x=a, y=0, y=b$. 5

(b) Find the surface integral $\iint \mathbf{F} \cdot \mathbf{n} dA$, where $\mathbf{F} = [x - z, y - x, z - y]$
 $\mathbf{S}: \mathbf{r} = [u \cos v, u \sin v, u], 0 \leq u \leq 3, 0 \leq v \leq 2\pi$. 5

