Registration No. : | | | | | | | | | | |

Total number of printed pages – 3

B. Tech
BS 1104

First Year Special Examination – 2014

MATHEMATICS – II

BRANCH(S) : AEIE, AUTO, BIOTECH, CHEM, CIVIL, CSE, EC, EEE, ELECTRICAL, ENV, ETC, FASHION, IEE, IT, MECH, MM, MME,TEXTILE

QUESTION CODE : G 613

Full Marks – 70

Time – 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

   (a) Define Dirac's delta function.

   (b) Find the convolution of Sinùt*cosùt

   (c) Write the formula for Fourier transform of derivatives.

   (d) What is Fourier integral representation ?

   (e) If $f = xy - yz$, then find grad (f^2)

   (f) Define vector field and scalar field.

   (g) What is the geometrical significance of curl ?

   (h) State Laplace's equation and its importance.

   (i) Evaluate $\int_0^2 \int_y^{y^2} (x^2y + 3x)dx$

P.T.O.

(j) How can we convert line integrals to surface integrals ? State the conditions in the corresponding theorem.

2. (a) Solve the given initial value problem by Laplace transform

$Y'' - 2y' + 2y = 8\,e^{-t}\cos t$, $y(0) = 16$, $y'(0) = -16$ 5

(b) Find the inverse Laplace transform of the given function 5

(i) $\frac{s+1}{s^2}e^{-s}$

(ii) $\frac{3s+4}{s^2+4s+5}$

3. (a) Solve the integral equation $y = 2t - 4\int_0^t y(\tau)(t-\tau)d\tau$ using Laplac Transform. 5

(b) Find Fourier series expansion of $f(x) = \cos ax$ in the range 0 to 2Π . 5

4. (a) Find Fourier transform of $f(x) = e^{-x}$ if $x \square 0$ and 0 if $x < 0$ 5

(b) Find the Fourier cosine integral representation of 5

$$f(x) = \begin{cases} x^2, & 0 < x < 1 \\ 0 & x > 1 \end{cases}$$

5. (a) Find the directional derivative of $f = 3xyz - xy^2$ at (1,2,2) in the direction of normal to the surface $x^2 + y^2 - z^2 = 1$ at (1,1,1). 5

(b) Prove that div ($\mathbf{u} \times \mathbf{v}$) = $\mathbf{v}$.curl $\mathbf{u}$ – $\mathbf{u}$ curl $\mathbf{v}$. 5

6. (a) Find a tangent vector and its corresponding unit tangent vector of the curve $r(t) = \cos t\,\mathbf{i} + 2\sin t\,\mathbf{j}$ at the point $(1/2, \sqrt{3}, 0)$ 5

(b) Find the scalar potential of $\mathbf{V} = [ye^x, e^x, 1]$ 5

7. (a) Determine whether the line integral is $\int \cos(x + yz)(dx + zdy + ydz)$ independent of the path or not . If so, then evaluate it from (1,0,1) to $(0, \Pi/2, 1)$. 5

(b) Find the moment of inertia of a lamina S : $x^2 + y^2 = z^2, 0 \le z \le h$ of density 1 about the z-axis. 5

8. (a) Verify Stokes theorem for a vector field defind by $\mathbf{F} = (x^2 - y^2)\,\mathbf{i} + 2xy\,\mathbf{j}$ in the rectangular region in the xy-plane bounded by the lines x= 0, x = a, y = 0, y = b. 5

(b) Find the surface integral $\iint F.ndA$, where $\mathbf{F} = [\,x + z\,,\ y - x\ ,\ z - y]$ **S**: $\mathbf{r} = [u \cos v,\ u \sin v\ ,\ u],\ 0 \le u \le 3, 0 \le v \le 2\Pi$. 5
