Registration No.:

Total number of printed pages - 2

B. Tech BS 1101

First Year Special Examination – 2014 MATHEMATICS – I

BRANCH(S): AEIE, AUTO, BIOTECH, CHEM, CIVIL, CSE, EC, EEE, ELECTRICAL, ENV, ETC, FASHION, IEE, IT, MANUFACT, MECH, MM, MME, PLASTIC, TEXTILE

QUESTION CODE: G 354

Full Marks - 70

Time - 3 Hours

Answer Question No. 1 which is compulsory and any five from the rest.

The figures in the right-hand margin indicate marks.

Answer the following questions :

2×10

(a) What is the order and degree of the following differential equation:

$$\left(1 + \frac{d^2y}{dx^2}\right)^3 = \frac{dy}{dx}$$

- (b) Find whether the vectors (1 1 3), (5 2 4), (3 2 6) are linearly independent or not.
- (c) Write the formula for radius of curvature in pedal form.
- (d) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^{n-1}}{(n+1)}$
- (e) Define Bessel's function of 2nd kind.
- (f) What is the value of $p_{2n+1}(0)$, the Legender polynomial of degree 2n+1.
- (g) Define Eigen value and Eigen vector of a matrix.
- (h) Define orthogonal matrix, unitary matrix.
- (i) Find the symmetric coefficient matrix of the quadratic form

$$Q = 2x_1^2 + 4x_1x_2 + 4x_2x_3 - 9x_3^2.$$

(j) What is algebraic multiplicity and geometric multiplicity of an eigen value.

- 2. (a) Solve the Bernoulli's equation $y'-2xy=2xy^2$. 5
 - (b) Solve the initial value problem $ye^x dx + (2y + e^x) dy = 0$, y(0) = -1.
- 3. (a) Solve: $(D^4 3D^2 4)y = 5\sin 2x e^{-2x}$.
 - (b) Solve: $\frac{d^2y}{dx^2}$ + 9y = sec3x using variation of parameter. 5
- (a) Solve: y" + 4y = 8x² using undetermined coefficient.
 - (b) Reduce the differential equation xy"+5y'+xy=0 to bessel's function. 5
- 5. (a) Find a power series solution of the differential equation y''-3y'+2y=0. 5
 - (b) Prove that $\int J_{v-1}(x) dx = \int J_{v-1}(x) dx 2J_v(x)$ where $J_n(x)$ is the Bessel's function of order n.
- 6. (a) Find the asymptotes of the cubic curve

$$2x^3 - x^2y + 2xy^3 + y^3 - 4x^3 + 8xy - 4x + 1 = 0.$$

(b) Show that the radius curvature at a point of the curve

$$x = ae^{\theta} (\sin \theta - \cos \theta), y = ae^{\theta} (\sin \theta + \cos \theta)$$

is twice the distance of the tangent at the point from the origin.

7. (a) Solve the following system of equations 4y+3z=8

$$2x - z = 2$$
$$3x + 2y = 5$$
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- (b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & -1 & 2 \\ 2 & 4 & 11 \end{bmatrix}$ using Gauss-jordan method.
- 8. (a) Show that the eigen value of a hermitian matrix are real. 5
 - (b) Diagonalize the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ and hence find $x^{-1}A^2x$.

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