Registration No. :

First Semester Back Examination – 2014

MATHEMATICS - I

BRANCH (S) : AEIE, AERO, AUTO, BIOMED, BIOTECH, CHEM, CIVIL, CSE, EC, EEE, EIE, ELECTRICAL, ENV, ETC, FASHION, FAT, IEE, IT, MANUFACT, MANUTECH, MECH, MINERAL, MINING, MM, MME, PLASTIC, TEXTILE

QUESTION CODE : L 347

Full Marks – 70

Time : 3 Hours

*Answer Question No. 1 which is compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions :   2×10
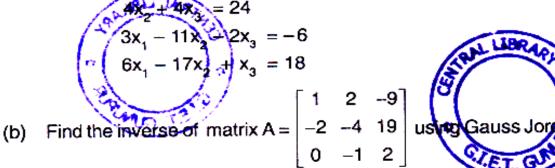
   (a) Define order and degree of a differential equation.

   (b) Explain method of variation to solve differential equation.

   (c) Does every first order ODE have a solution ? What do you know about uniqueness of solutions ?

   (d) What do you mean by linear independent or dependent function ? Give example of three linearly independent functions.

   (e) Explain Legender polynomial.

   (f) Find the radius of convergence of the power series $\sum\limits_{n=1}^{\infty} \dfrac{x^{2n+10}}{(2n+1)\,n}$.

   (g) Define curvature.

   (h) Prove that if A is a non-singular square matrix, then A^{-1} is also nonsingular.

   (i) Define similar matrix, orthonormal matrix.

   (j) Define rank of a matrix.

2. (a) Solve $(x - 2y)\, dy = (x - 2y + 1)\, dx$.   5

   (b) Solve $x^2 y'' + 3xy' + y = 0$.   5

3. (a) Solve the initial value problem $x^2y'' + 3xy' + y = 0$, $y(1) = 4$, $y'(1) = -2$.   5

(b) Solve $y'' + 4y + 4y = e^{-3x} \sin 2x$ using undetermined coefficient.   5

4. (a) Solve $\dfrac{d^2y}{dx^2} + 4y = \cosh 2x$ using variation of parameter.   5

(b) Find the general solution of $(4D^2 + 12D + 13I)y = 11\cos 5t$, $y(0) = 1$, $y'(0) = 1$.   5

5. (a) Find a power series solution of the differential equation $y'' - 3y' + 2y = 0$.   5

(b) Find the solution of the following equation in terms of Bessel function :   5

$$4y'' + 9xy = 0.$$

6. (a) Find the asymptotes of the curve $Y^2(x - 2a) = x^3 - a^3$.   5

(b) Show that the radius curvature of the curve $x^3 + y^3 = 3axy$.   5

7. (a) Solve the system of equations using Gauss elimination method :   5

$$4x_1 + 4x_2 + 4x_3 = 24$$
$$3x_1 - 11x_2 - 2x_3 = -6$$
$$6x_1 - 17x_2 + x_3 = 18$$

(b) Find the inverse of matrix $A = \begin{bmatrix} 1 & 2 & -9 \\ -2 & -4 & 19 \\ 0 & -1 & 2 \end{bmatrix}$ using Gauss Jordan method.

   5

8. (a) Find a basis of the eigen vectors and diagonalize the following matrix :   5

$$\begin{bmatrix} -14 & 10 \\ -10 & 11 \end{bmatrix}.$$

(b) Find out what type of conic section the following quadratic form represents and transform it to principal axes :   5

$$x_1^2 - 12x_1x_2 + x_2^2 = 35.$$