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Total Number of Pages: 2

**B.TECH**  
**BS1101**

## 1<sup>st</sup> Semester Back Examination 2015-16

### MATHEMATICS - I

BRANCH: All (Back)

Time: 3 Hours

Max Marks: 70

Q.CODE: T806

**Answer Question No.1 which is compulsory and any five from the rest.  
The figures in the right hand margin indicate marks.**

- Q1** Answer the following questions: **(2 x 10)**
- a) Find the radius of curvature of  $y = e^x$  at (0,1).
  - b) Find all the asymptotes to the curve  $x^2 y^2 - x^2 y - xy^2 + x + y + 1 = 0$  which are parallel to the axes.
  - c) Find an integrating factor for the ordinary differential equation  $(x^2 + y^2 + x)dx + xydy = 0$ .
  - d) Solve the ordinary differential equation  $y' = xy$
  - e) Construct the homogeneous ordinary differential equation with constant coefficient whose independent solutions are  $e^x$  and  $xe^x$ .
  - f) Are the functions  $x|x|$ ,  $x^2$  linearly independent in the interval  $0 \leq x \leq 1$ ? Justify your answer.
  - g) Find the radius of convergence of the power series  $\sum_{m=0}^{\infty} (m+1)mx^m$
  - h) If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix}$  then find  $A^T B$ .
  - i) Find the eigen values of the matrix  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ .
  - j) Show that the determinant of an orthogonal matrix has values +1 or -1
- Q2** a) Show that  $3\sqrt{3}/2$  is the least value of  $|\rho|$  for  $y = \log x$ . **(5)**  
b) Find all the asymptotes of the curve **(5)**  
 $x^4 y + 2x^3 y^2 - x^2 y^3 - 2xy^4 - x^3 y + xy^3 + x^2 + y^2 + 1 = 0$
- Q3** a) A thermo meter, reading  $15^\circ\text{C}$ , is brought into a room whose temperature is  $30^\circ\text{C}$ . One minute later the thermo meter reading is  $22^\circ\text{C}$ . How long does it take until the reading is  $29.9^\circ\text{C}$ ? **(5)**  
b) Solve the ordinary differential equation  $(x + 2y)dx + (2x + y)dy = 0$ . **(5)**

**Q4 a)** Solve  $y'' + 4y' + 4y = 0$ ,  $y(0) = 1$  and  $y'(0) = 1$ . (5)

**b)** Find the curve through the origin in the  $xy$ -plane which satisfies  $y'' = 2y'$  and whose normal at the origin has slope  $-1$ . (5)

**Q5 a)** Solve  $y'' - 3y' + 2y = 4x^2$ . (5)

**b)** Find a power series solution of  $y' + 2xy = 0$  in powers of  $x$ . Show the details of your work. (5)

**Q6 a)** Show that  $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ . (5)

**b)** Solve  $y'' + x^2 y = 0$  using the substitution  $y = u\sqrt{x}$ ,  $\frac{x^2}{2} = z$ . (5)

**Q7 a)** Solve the given system of linear equation (5)  
 $x + 2y + z - w = 0$ ,  $2x + y + 3z + w = 1$ ,  $-3x - y + 2z + 3w = 0$

**b)** Show that the set of all vectors  $(v_1, v_2, v_3)$  in  $R^3$  such that  $2v_1 + 3v_3 = 0$  is a vector space, determine its dimension and a basis. (5)

**Q8 a)** Find all the eigen values and the eigen vector corresponding to the smallest eigen value for the matrix  $T$  as given below (5)

$$T = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

**b)** Show that the eigen values of a Hermitian matrix are real. (5)