Registration No.: | | | | | | | | | | |

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B.Tech.
15BS1101

First Semester Regular Examination, 2015-16

Mathematics - I

BRANCH(S) : All

Time : 3 Hours
Max. Marks : 100

Q. CODE : T805

Answer Part-A which is compulsory and any four from Part-B.
The figures in the right-hand margin indicate marks.

Part-A (Answer all the questions)

1. Answer the following questions. [2 × 10]

   (a) The eigen-vectors of standard matrices $O_{n\times n}$ and $I_{n\times n}$ are (same/different).

   (b) The algebraic multiplicity is (grater/smaller) than the geometric multiplicity of an eigen-value.

   (c) The geometric multiplicity of $n\times n$ identity matrix is ———————————————————.

   (d) The sum of the series $\sum\limits_{n=2}^{\infty} \frac{1}{n(n-1)}$ is ——————————————————————————.

   (e) Solutions $y_1(x) = |x|x$ and $y_2(x) = x^2$ are (independent/dependent) over the interval $(-\infty, 0)$.

   (f) The general solution of $y' + \ln(x)y = 0$ is ————————————————————————.

   (g) The general for of particular solution of the equation $y'' = x$ is —————————————.

   (h) If $y(x) = ae^{x} + be^{-x}$ is satisfying the condition $y(0) = 0$ and $y'(0) = 0$, then the parameters are $a =$ ————————————————— and $b =$ ———————————————————.

   (i) The number of inclined asymptotes to any algebraic curve of degree n is ——————————.

   (j) Curvature of a circle having radius a is —————————————————————————.

2. Answer the following questions. [2 × 10]

   (a) Find the general solution of $xy' + y = 2xe^{x^2}$.

   (b) Find the integrating factor of the equation $(xy - y)\,\mathrm{d}x + (xy - x)\mathrm{d}y = 0$.

   (c) Reduce the equation $2yy' - 2xy^2 = x$ to linear differential equation.

   (d) Find the Wronskian of the differential equation $y'' + xy' + y = x$.

   (e) Find the second independent solution of the equation $y'' + xy' - y = 0$ using $y_1(x) = x$ as first solution.

   (f) Find the series representation of $\frac{1}{(1-x)^n}$ about $x = 0$.

   (g) Find the dependency of the vectors (1, 0, 3), (0, 1, 2), (2, 3, 1) and (4, 1, 0).

   (h) Express a square matrix as sum of symmetric and skew-symmetric matrices.

(i) Find all independent eigen-vectors of 3×3 identity matrix.

(j) If $|\lambda I - A| = \lambda^n + \sum_{i=0}^{n-1} \alpha_i \lambda^i$, then find the determinant $|A|$.

<u>PART-B</u> (Answer any four questions)

3. Answer in detail.
   (a) Derive the solution of the equation $y' - p(x)y = \phi(x)y^n$ for $n \geq 2$. [10]
   (b) Solve the differential equation $y(x^3 - y)\mathrm{d}x - x(x^3 + y)\mathrm{d}y = 0$. [5]

4. Answer according to instruction.
   (a) If $y_1(x)$ and $y_2(x)$ are two independent homogeneous solutions of the differential equation $r(x)y'' + p(x)y' + q(x)y = \phi(x)$, then derive the formula for general solution. [10]
   (b) Find the solution of the differential equation $y'' - 3y' + 2y = \sin(e^{-x})$. [5]

5. Answer according to requirement.
   (a) Find the series solution $y'' - 5y' + 4y = 0$ about $x = 0$ and hence deduce the closed form solution. [10]
   (b) Find series solution of the equation $(1 - x)y' - y = 0$ about $x = 0$. [5]

6. Give answer in detail.
   (a) Show that $J_{-\frac{1}{2}}(x) = \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \cos(x)$. [10]
   (b) Show that $mP_{m-1}(x) + (m+1)P_{m+1}(x) - (2m+1)xP_m(x) = 0$. [5]

7. Give detail derivation.
   (a) Derive transforms for x and y so that the quadratic equation $9x^2 + 6xy + y^2 = 40$ transferred to canonical form. Find the nature and identify the name of the canonical form. [10]
   (b) Show that eigen-vectors of symmetric matrix corresponding to different eigen-values are orthogonal. [5]

8. Give detail answer.
   (a) State the conditions under which a system of equations to have unique solution, infinite solutions and no solution. Solve the system of equations $x + 3y - 3z = 1$, $x + y + 2y = 4$ and $2x + 2y - z = 3$. [10]
   (b) Find the three independent eigen-vectors of the matrix [5]

$$\begin{pmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{pmatrix}$$

9. Answer in detail.
   (a) Show that the radius of curvature at any point of the curve $x = a\cos^3(\theta)$ and $y = a\sin^3(\theta)$ is equal to three times the length of the perpendicular from origin to the tangent at that point. [10]
   (b) Find all asymptotes to the algebraic curve $x^2y + xy^2 + xy + y^2 + 3x = 0$. [5]

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