2013

Time : 4 hours

Full Marks : 100

The questions are of equal value.

*Answer any **five** questions.*

Symbols used have their usual meanings.

(OPERATIONS RESEARCH)

1. Using Gomory's algorithm solve the following Integer programming problem :

   Maximize $Z = 9x_1 + 10x_2$

   Subject to the conditions $x_1 + x_3 = 3$; $2x_1 + 5x_2 + x_4 = 15$ and $x_1, x_2, x_3, x_4 \geq 0$ and integers.

2. (a) Find a point (x, y, z) on the surface $x^2 - z^2 = 1$ that is nearest to origin.

   (b) Let f be a real-valued function defined on a set $T \subset R^n$, where T is a connected open or

(b) Show that a point $x^o \in \Omega p$ is an optimal solution to program QP, if and only if, for some $\lambda^o \geq 0$, (x^o, λ^o) is a saddle point of the Lagrangian $L(x, \lambda) = f(x) - \lambda^T(Ax - b)$, where $\lambda \geq 0$.

7. (a) Solve the following program using the Frank and Wolfe algorithm :

Minimize $f(x) = x_1^2 + 4x_2^2$

subject to :

$x_1 + 2x_2 - x_3 = 1$; $-x_1 + x_2 + x_4 = 0$; $x_i \geq 0$, $(i = 1, 2, 3, 4)$

(b) Using Kelley's cutting plane algorithm solve :

Minimize $f(x) = x_2 - x_1$

Subject to :

$x_1 + x_2 \leq 5$; $x_1 x_2 \geq 4$; $x_1 \geq 0$, $x_2 \geq 0$.

8. Show that the logarithm of the dual function

$$\ln v(\delta) = \sum_{k=0}^{m}\left(\sum_{i=1}^{T_K}\delta_{ki}\right)\ln\left(\sum_{i=1}^{T_K}\delta_{ki}\right) - \sum_{k=0}^{m}\sum_{i=1}^{T_k}\delta_{ki}$$

$$\ln(\delta_{ki}) + \sum_{k=0}^{m}\sum_{i=1}^{T_k}\delta_{ki}\ \ln(c_{ki})$$ is strictly concave

for $\delta > 0$.

9. (a) Using Dynamic programming find the height attained by a projectile.

(b) Explain principle of optimality on dynamic programming problem.

10. (a) Show that every symmetric game has the value $\nu = 0$ and each player has the same set of optimal strategies.

(b) Using Dominance principle solve the 4 ×3 game with the pay-off matrix

$$A = \begin{pmatrix} 8 & 5 & 8 \\ 8 & 6 & 5 \\ 7 & 4 & 5 \\ 6 & 5 & 6 \end{pmatrix}.$$

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