

2013

Time : 4 hours

Full Marks : 100

The questions are of equal value.

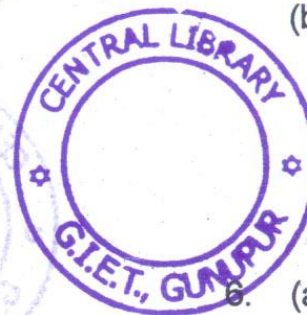
Answer any five questions.

(Matrix Transformations in Sequence Space)

1. (a) Show that :
 - (i) The Nörlund mean (N, p_n) is regular iff $\frac{p_n}{P_n} \rightarrow 0$, as $n \rightarrow \infty$.
 - (ii) The Riesz mean (R, p_n) is regular iff $P_n \rightarrow \infty$, as $n \rightarrow \infty$.
- (b) Define a limitation method and show that the limitation method defined by $t_n = \frac{s_1 + s_2 + \dots + s_n}{n}$ is regular.
2. (a) Show that a matrix cannot be both regular and a Schur matrix.

8. (a) If $A = (a_{mn})$ and $B = (b_{mn})$ are regular matrices and $A \supseteq B$. Then show that $\|A\| \geq \|B\|$.
- (b) Show that a regular matrix $A = (a_{mn})$ limits a sequence in the unit ball $\{s_n^1\}$ such that $N(A) = A - \lim s_n^1$.
9. (a) Let A be any bounded linear operator C_0 into itself. Then show that A determines a matrix (a_{nk}) such that $(Ax)_n = \sum a_{nk} x_k$ for every $x \in C_0$ and such that $\|A\| = \sup \sum |a_{nk}| < \infty$, $a_{nk} \rightarrow 0$ as $n \rightarrow \infty$.
- (b) Let $1 < p < \infty$ and suppose $A \in (l_\infty, l_\infty) \cap (l_1, l_1)$. Then show that $A \in (l_p, l_p)$.
10. (a) State and prove Silverman – Toeplitz Theorem.
- (b) Let m and M be constants such that $0 < m \leq p_k \leq M$. Then show that (A, p) is strongly regular if and only if $A \in (C_0, C_0)$.





(b) Show that to every bounded sequence σ there corresponds a matrix A of the class P such that the A -limit of σ exists.

3. (a) Show that, if (N, p_n) is a regular Nörlund method, then the series $\sum_{n=1}^{\infty} p_n x^{n-1}$ and

$$\sum_{n=1}^{\infty} P_n x^{n-1}$$

are convergent for all $|x| < 1$.

(b) Prove that for every positive integer k , the (C, k) and (H, k) matrices are equivalent.

4. (a) If $\alpha > 0$, show that the transformation t_n represented by $t_n = \alpha s_n + (1 - \alpha) \frac{s_0 + s_1 + \dots + s_n}{n + 1}$ is equivalent to convergence.

(b) Show that (C, k) method is a Hausdorff method corresponding to the function :

$$\phi(x) = k \int_0^x (1-t)^{k-1} dt$$

5. (a) Show that the sequence $\{s_n\}$ is almost convergent if and only if $P(s_n) = -P(-s_n)$.

(b) Show that the necessary and sufficient condition for a regular matrix $A = (a_{mn})$ to have a counting function of the first kind is

$$\lim_{m \rightarrow \infty} \max_n |a_{mn}| = 0$$

6. (a) Let $A = (a_{mn})$ and $B = (b_{mn})$ be regular triangular matrices. If A is a perfect matrix and $O(A) \subseteq O(B)$, then show that B is a stronger than A .

(b) Show that no regular matrix limits all sequences of 0's and 1's.

7. (a) If $A = (a_{mn})$ and $B = (b_{mn})$ are regular matrices and A is μ_n -stronger than B , then show that A and B are ρ_n -consistent for some $\rho_n \rightarrow \infty (\rho_n \leq \mu_n)$.

(b) Define a truncated matrix (and show that for a regular matrix) $A = (a_{mn})$ there exists a sequence $\{\rho_n\}$, $\rho_n \rightarrow \infty$ and a truncated matrix $B = (b_{mn})$ such that

$$\lim_{m \rightarrow \infty} \left(\sum_{n=1}^{\infty} a_{mn} s_n - \sum_{n=1}^{\infty} b_{mn} s_n \right) = 0$$

for every $\{s_n\}$, $s_n = O(\rho_n)$.