8. (a) If $A = (a_{mn})$ and $B = (b_{mn})$ are regular matrices and $A \supseteq B$. Then show that $\| A \| \geq \| B \|$.

(b) Show that a regular matrix $A = (a_{mn})$ limits a sequence in the unit ball $\{ s_n^1 \}$ such that $N(A) = A - \lim s_n^1$.

9. (a) Let A be any bounded linear operator C_o into itself. Then show that A determines a matrix (a_{nk}) such that $(A_x)_n = \Sigma a_{nk} x_k$ for every $x \in C_o$ and such that $\| A \| = \sup \Sigma | a_{nk} | < \infty$, $a_{nk} \to 0$ as $n \to \alpha$.

(b) Let $1 < p < \infty$ and suppose $A \in (l_\infty, l_\infty) \cap (l_1, l_1)$. Then show that $A \in (l_p, l_p)$.

10. (a) State and prove Silverman – Toeplitz Theorem.

(b) Let m and M be constants such that $0 < m \leq p_k \leq M$. Then show that (A, p) is strongly regular if and only if $A \in (C_o, C_o)$.

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2013

Time : 4 hours

Full Marks : 100

The questions are of equal value.

Answer any ***five*** *questions.*

(Matrix Transformations in Sequence Space)

1. (a) Show that :

(i) The Nörlund mean (N, p_n) is regular iff $\frac{p_n}{P_n} \to 0$, as $n \to \infty$.

(ii) The Riesz mean (R, p_n) is regular iff $P_n \to \infty$, as $n \to \infty$.

(b) Define a limitation method and show that the limitation method defined by $t_n = \frac{s_1 + s_2 + \ldots\ldots\ldots + s_n}{n}$ is regular.

2. (a) Show that a matrix cannot be both regular and a Schur matrix.

(b) Show that to every bounded sequence σ there corresponds a matrix A of the class P such that the A-limit of σ exists.

3. (a) Show that, if (N, p_n) is a regular Nörlund method, then the series $\sum_{n=1}^{\infty} p_n x^{n-1}$ and $\sum_{n=1}^{\infty} P_n x^{n-1}$ are convergent for all $|x| < 1$.

(b) Prove that for every positive integer k, the (C, k) and (H, k) matrices are equivalent.

4. (a) If $\alpha > 0$, show that the transformation t_n represented by $t_n = \alpha s_n + (1-\alpha)\dfrac{s_0 + s_1 + \cdots\cdots + s_n}{n+1}$ is equivalent to convergence.

(b) Show that (C, k) method is a Hausdorff method corresponding to the function :

$$\phi(x) = k\int_0^x (1-t)^{k-1}\,dt$$

5. (a) Show that the sequence $\{s_n\}$ is almost convergent if and only if $P(s_n) = -P(-s_n)$.

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(b) Show that the necessary and sufficient condition for a regular matrix $A = (a_{mn})$ to have a counting function of the first kind is $\lim_{m \to \infty} \max_n |a_{mn}| = 0$.

6. (a) Let $A = (a_{mn})$ and $B = (b_{mn})$ be regular triangular matrices. If A is a perfect matrix and $O(A) \subseteq O(B)$, then show that B is a-stronger than A.

(b) Show that no regular matrix limits all sequences of 0's and 1's.

7. (a) If $A = (a_{mn})$ and $B = (b_{mn})$ are regular matrices and A is μ_n-stronger than B, then show that A and B are ρ_n-consistent for some $\rho_{n \to \infty} (\rho_n \le \mu_n)$.

(b) Define a truncated matrix (and show that for a regular matrix) $A = (a_{mn})$ there exists a sequence $\{\rho_n\}$, $\rho_{n \to \infty}$ and a truncated matrix $B = (b_{mn})$ such that

$$\lim_{m \to \infty}\left(\sum_{n=1}^{\infty} a_{mn}s_n - \sum_{n=1}^{\infty} b_{mn}s_n\right) = 0 \text{ for}$$

every $\{s_n\}$, $s_n = O(\rho_n)$.