26. Show that R^n is homeomorphic to R^m iff $m = n$.

27. Let α_0 and α_1, be paths in a space X with $\alpha_0 \overset{p}{\simeq} \alpha_1$. Then show that $\alpha_0^{-1} \overset{p}{\simeq} \alpha_1^{-1}$.

28. If (X, x_0) and (Y, y_0) are pointed spaces and $f : (X, x_0) \rightarrow (Y, y_0)$ is a homotopy equivalence. Then show that the induced map $f_* : \pi_1 (X, x_0) \rightarrow \pi_1 (Y, y_0)$ is an isomorphism.

29. Show that for any oriented complex K and any $p \geq 0$, The composition

$$C_{p+1}(K) \rightarrow C_p(K) \rightarrow C_{p-1}(K)$$

is a trivial homomorphism.

30. Let K be an oriented complex and $\overline{K}$ its augmented complex. Then show that $H_p(\overline{K}) \approx H_p(K)$ for all $p > 0$ and $H_0(K) = H_0(\overline{K}) \oplus Z$.

❖

2013

Time : 4 hours

Full Marks : 100

The questions are of equal value.

*Answer any **five** questions each from any **two** Groups as per your Course.*

Group – A

(Graph Theory)

1. Show that for any graph G with e edges and n vertices $v_1, v_2, \ldots, v_n$

$$\sum_{i=1}^{n} d(v_i) = \alpha e$$

Further show that in any graph G there is an even number of odd vertices.

2. Let G be a graph with n vertices $v_1, v_2, \ldots, v_n$ and let A denotes the adjacency matrix of G with respect to this listing of the vertices. Let $B = (b_{ij})$ be the matrix.

$$B = A + A^2 + \cdots + A^{n-1}$$

Then show that G is connected if and only if for every pair of distinct indices i, j we have $b_{ij} \neq 0$.

3. Show that a Tree with n vertices has precisely n–1 edges.

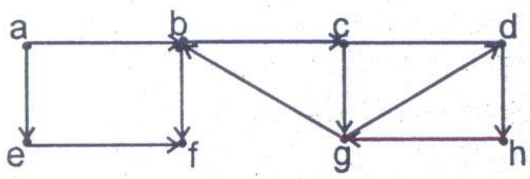
4. Prove that a connected graph G is Euler if and only if the degree of every vertex is even.

5. Show that a complete graph on five vertices is non-planar.

6. If G is a simple plannar graph then show that G has a vertex v of degree less than 6.

7. Show that for any graph G, $\chi(G) \leq \Delta(G) + 1$.

8. Find od(v) and id (v) for each vertex of the diagraph :



9. Let u and v two distinct vertices of the graph G. Then prove that a set S of vertices of G is u – v separating iff every u – v path has at least one internal vertex belonging to S.

10. Define :
    (a) Subgraph
    (b) Bridge
    (c) Euler graph
    (d) Hamiltonian graph
    (e) Chromatic number

Group – B
(PROGRAMMING IN C)

11. (a) Determine the hierarchy of operations and hence evaluate the expression 3/2 * 4+ 3/8+3.

    (b) Convert the equation $X = \frac{-b + (b * b) + 2/(4ac)}{2a}$ into the corresponding C statement.

12. Write a program using conditional operators to determine whether a year entered through the keyboard is a leap year or not.

13. Explain, with example, the difference between While Loop and For Loop.

14. Write a program to find factorial value of any member entered through the keyboard.

15. Write a recursive function in C to obtain the first 25 members of a Fibonacci sequence.

16. Write short notes on the following :

    (a) Use of functions in C program

    (b) Passing values between functions

17. Explain, with example, the following :

    (a) Long and short integers

    (b) Floats and doubles

    (c) Signed and unsigned integers

18. If a macro is not getting expanded as per your expectation, explain how will you find out how is it being expanded by the preprocessor.

19. (a) Using array write a program to find average marks obtained by a class of 30 students in a test.

    (b) Write down a program to pick up the largest number from any 5 row by 5 column matrix.

20. The X and Y coordinates of 10 different points are entered through the keyboard. Write a program to find the distance of last point from the first point (sum of the distances between consecutive points).

Group – C

(ALGEBRAIC TOPOLOGY)

21. Show that a retract of a space X is necessarily closed in X.

22. If $s_p = \{a_1, a_2, .., a_p\} \subseteq R^n$ is geometrically independent then show that $\overline{s}_p$ is the closed convex hull of s.

23. Let K and L be geometric complexes and $f : |K| \rightarrow |L|$ a continuous map. Show that simplicial map $\phi : |K| \rightarrow |L|$ is a simplicial approximation to f iff for each $a \in K_a$, $f(st(a)) \subseteq st(\phi(a))$.

24. Let K and L be geometric complexes ; and $f : |K| \rightarrow |L|$ a continuous map and $\phi : |K^{(m)}| \rightarrow |L|$ and $\psi : |K^{(m)}| \rightarrow |L|$ two simplicial approximations to f. Then show that ϕ and ψ are contiguous.

25. Show that the following statements are equivalent :

    (a) S^{n-1} is not a retract of B^x.

    (b) B^n has the fixed point property.