2014

Time : 4 hours

Full Marks : 100

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The questions are of equal value.

Answer any ***five*** *questions from each Group.*

Symbols used have their usual meanings.

(LINEAR PROGRAMMING AND MEASURE AND INTEGRATION)

Group – A

Marks : 50

(Linear Programming)

1. Define basic feasible solution of the Linear Programming Problem. Find all basic feasible solutions for the equations :

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

(Turn over)

2. If $X = (x_1, x_2, \ldots, x_n)$ is an extreme point of K, then show that the vectors associated with positive x_i form a linearly independent set.

3. Prove that if for any basic feasible solution $X = (x_{10}, \ldots, x_{m0})$. The condition $q_i - c_i \leq 0$ hold for all j = 1, 2,...., n, then it is a minimum feasible solution of the LPP.

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4. Using Simplex Method :

Minimize $x_2 - 3x_3 + 2x_5$

Subject to the conditions :

$x_1 + 3x_2 - x_3 + 2x_5 = 7$

$-2x_2 + 4x_3 + x_4 = 12$

$-4x_2 + 3x_3 + 8x_5 + x_6 = 10$

$x_j \geq 0$, for j = 1, 2, 3, 4, 5, 6

5. Maximize : $-2x_1 - x_2 + x_3 + x_4$

Subject to the conditions :

$x_1 - x_2 + 2x_3 - x_4 = 2$

$2x_1 + x_2 - 3x_3 + x_4 = 6$

$x_1 + x_2 + x_3 + x_4 = 7$

and $x_j \geq 0$, for j = 1, 2, 3, 4

6. Using revised simplex method

Minimize : $-x_1 + 2x_2$

Subject to the conditions :

$5x_1 - 2x_2 \leq 3$

$x_1 + x_2 \geq 1$

$-3x_1 + x_2 \leq 3$

$-3x_1 - 3x_2 \leq 2$

$x_1 \geq 0$ and $x_2 \geq 0$

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7. State and prove complementary slackness theorem in duality.

8. Find the dual of the problem :

Minimize : $2x_1 - 3x_2$

Subject to the conditions :

$2x_1 - x_2 - x_3 \geq 3$

$x_1 - x_2 + x_3 \geq 2$

$x_j \geq 0$, for j = 1, 2, 3.

9. What do you mean by a transportation problem ? Write the transportation problem in standard firm and show that it has a feasible solution.

10. Solve the transportation problem :

1	2	3	4	6
4	3	2	0	8
0	2	2	1	10
4	6	8	6	

Group – B

Marks : 50

(Measure and Integration)

11. Let $\{A_n\}$ be a countable collection of sets of real numbers. Then show that :

$$m^* (UA_n) \leq \Sigma\, m^* A_n$$

12. Construct a non-measurable set.

13. Define a measurable function and show tha if $< f_n >$ is a sequence of measurable functions with the same domain, then $\underline{\lim} f_n$ and $\overline{\lim} f_x$ are also measurable.

14. If $< f_n >$ is a sequence of measurable functions (with the same domain) that converges to a real-valued function f a.e on a measurable set E of finite measure, then show that for a given $\eta > 0$ there is a subset $A \subset E$ with $m\, A < \eta$ such that f_n converges to f uniformly on $E \sim A$.

15. State and prove Lebesque convergence theorem.

16. Let f be a bounded function defined on [a, b]. If f is Riemann integrable or [a, b]. Then show that f is measurable and also Lebesque integrable.

17. Define a function of bounded variation and show that a function f is of bounded variation on [a, b] if and only if f is the difference of two monotone real valued functions on [a, b].

18. Show that if f is absolutely continuous and $f'(x) = 0$ a.e, then f is constant.

19. State and prove Minkowski Inequality.

20. Show that every convergent sequence is a Cauchy sequence.

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