with h = 0.2 on the interval [0, 0.4], using Euler – Cauchy method.

20. Write short notes on :

(i) Runge – Kutta Method

(ii) Interpolation

2014

Time : 4 hours

Full Marks : 100

The questions are of equal value.

*Answer any **five** questions from each Group.*

Symbols used have their usual meanings.

(GENERAL TOPOLOGY AND NUMERICAL ANALYSIS)

Group – A

Marks : 50

(GENERAL TOPOLOGY)

1. Show that J* is a topology for X*.

2. Show that if F is a closed set then CF is an open set and conversely if CF is an open set then F is a closed set.

3. Define finite intersection property and show that a topological space (X, J) is compact if and only if any family of closed sets having the finite intersection property has a non-empty intersection.

4. Show that :

(a) If C is a connected set and $C \subseteq E \subseteq c(C)$, then E is connected.

(b) If every two points of a set E are contained in some connected subset of E, then E is a connected set.

5. Show that every compact subset E of a Hausdorff space X is closed.

6. Prove that in a second axiom space every collection of non-empty disjoint open sets are countable.

7. Show that a topological space X is completely normal if and only if every subspace of X is normal.

8. Show that normality is a topological property.

9. Show that X × Y is compact if and only if X and Y are compact.

10. Prove that the projections π_x and π_y are continuous and open mappings and so the product topology is the smallest topology for which the projections are continuous.

Group – B

Marks : 50

(NUMERICAL ANALYSIS)

11. For the data :

x	f(x)
0	1
1	2
2	33
3	244

fit a quadratic splines with $M(0) = f''(0) = 0$ Hence find an estimate of f(2.5).

12. For the data :

x	f(x)
0.1	1.40
0.2	1.56
0.3	1.76
0.4	2.00
0.5	2.28

Obtain the forward and backward difference polynomials.

13. Use the method of least squares to fit the curve $f(x) = C_0x + (C_1/\sqrt{x})$ for the data :

x	f(x)
0.2	16
0.3	14
0.5	11
1	6
2	3

Find also the least squares error.

14. Using Chebyshev polynomials, find nearly the best uniform approximation of degree 3 or less to x^4 on $[-1, 1]$.

15. Derive the formula for the first derivative of $y = f(x)$ of $0(h^2)$ using central difference approximation.

16. Evaluate $\int_0^{\infty} \frac{e^{-x}}{1+x^2}\, dx$ using Gauss-Leguerre two-point and three point formulas.

17. Find the approximate value of the integral $\int_0^1 \frac{dx}{1+x}$ using composite trapezoidal rule with 2, 3, 5, 9 nodes Romberg integration.

18. Solve the difference equation

$$\Delta^2 y_n + 3\Delta y_n - 4y_n = n^2$$

With the initial conditions $y_0 = 0$, $y_2 = 2$.

19. Solve the system of equations

$$u' = -3u + 2v,\ u(o) = 0$$

$$v' = 3u - 4v,\ v(o) = 0.5$$