9. (a) If V is n-dimensional over F and if $T \in (V)$ has all its characteristic roots in F, then show that T satisfies a polynomial of degree n over F.

(b) Show that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$.

10. (a) If $T \in A(V)$, then show that tr T is the sum of the characteristic roots of T.

(b) Show that if T is unitary and if λ is a characteristic root of T, then $|\lambda| = 1$.

2014

Time : 4 hours

Full Marks : 100

The questions are of equal value.

*Answer any **five** questions.*

Symbols used have their usual meanings.

(Algebra)

1. (a) Define permutation of n symbols and show that every permutation is the product of its cycles.

(b) Let G be a group and ϕ an automorphism of G. If $a \in G$ is of order $o(a) > 0$, then show that $o(\phi(a)) = o(a)$.

2. (a) If G is a group of order $o(G) = p^n$, where p is a prime number, then show that $Z(G) \neq (e)$.

(b) Show that every integral domain can be imbedded in a field.

3. (a) Show that $J[i]$ in a Euclidean Ring.

(b) If $f(x)$ and $g(x)$ are two non-zero elements of $F(x)$, then show that $\deg(f(x)g(x)) = \deg f(x) + \deg g(x)$.

4. (a) Let U and W be finite-dimensional subspaces of a vector space V. Then show that U + W is finite-dimensional and $\dim(U + W) = \dim U + \dim W - \dim(U \cap W)$.

(b) Define linear span of a subset of a vector space V and show that L (s) is a subspace of V.

5. (a) Show that a polynomial of degree n over a field can have atmost n roots in any extension field.

(b) Prove that a regular pentagon is constructible.

6. (a) Show that every finite-dimensional inner product space has an orthonormal set as basis.

(b) Show that $A(A(W)) = W$.

7. (a) If G is a group of automorphisms of K, then show that the fixed field of G is a subfield of K.

(b) If G is a group, prove that all G (k) are normal subgroups of G.

8. (a) If V is finite-dimensional over F, then show that $T \in A(v)$ is singular if and only if there exists a $v \neq 0$ in V such that $vT = 0$.

(b) Show that for the matrix :

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{pmatrix} \in F_3$$

There exists a matrix $C \in F_3$ such that

$$CAC^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$