is uniformly limitable to zero by B is also uniformly limitable to zero by A.

(b) Show that the norm of the bounded convergence field of a regular matrix is not always attained.

9. (a) Lets $a_{nk} \to 0$ and suppose $M = \sup_n \sum |a_{nk}| < \infty$ where sum is over k for each n. Then show that A defines a bounded linear operator on C_0 into itself and $||A|| = M$.

(b) Show that $A \in (c, c)$ if and only if :

(i) $\sup_n \sum_{k=1}^{\infty} |a_{nk}| < \infty$.

(ii) For each p, there exists $\lim_n \sum_{k=p}^{\infty} a_{nk} = a_p$.

10. (a) Show that a sequence is almost convergent if and only if $\lim_{n \to \infty} \frac{s_n + \cdots\cdots + s_{n+k-1}}{k} = s$, uniformly in n.

(b) Prove that the condition $\lim_{m \to \infty} \max_n |a_{mn}| = 0$, is necessary and sufficient for a regular matrix $A = (a_{mn})$ to have a counting function of the first kind.

2015

Time : 4 hours

Full Marks : 100

The questions are of equal value.

*Answer any **five** questions.*

The symbols used have their usual meanings.

(MATRIX TRANSFORMATION IN SEQUENCE SPACE)

1. (a) Define translative limitation matrix and show that the limitation matrix $A = (a_{m,\ n})$ is translative if and only if

$$\lim_{m \to \infty} \sum_{n=1}^{\infty} \left| a_{mn} - a_{m,\ n+1} \right| = 0.$$

(b) Let $A = (a_{mn})$. Then show that a necessary and sufficient condition for all the sequences of either class is that $\sum_{n=1}^{\nabla} |a_{mn}|$ converges for all m.

2. (a) Prove that the Nörlund mean (N, P_n) is regular if and only if $p_n/P_n \to 0$ as $n \to \infty$.

(b) Show that a regular matrix A is stronger than the Cesáro matrix if and only if there is a constant K such that $\sum_{n=1}^{\infty} n|a_{mn} - a_{m, n+1}| < K$ for all m.

3. (a) Show that for any two regular Nörlund means (N, p_n) and (N, q_n), there always exist a third mean (N, γ_n) such that $(N, \gamma_n) \supseteq (N, p_n)$ and $(N, \gamma_n) \supseteq (N, q_n)$.

(b) Show that the binomial coefficients satisfy the relation $\binom{n+k}{k} = \sum_{v=0}^{n} \binom{v+k-1}{k-1}$, $k = 1, 2, \cdots\cdots$.

4. (a) Show Abel's method of limitation is regular.

(b) Prove that the regular means (N, p_n) and (N, q_n) are equivalent if and only if the two associated series $\sum_{n=1}^{\infty} |k_n|$ and $\sum_{n=1}^{\infty} |h_n|$ converge.

5. (a) If (N, p_n) and (N, q_n) are regular Nörlund means, then show that $(N, p_n) \supseteq (N, q_n)$ if and only if there is an M such that for every $n|k_1| P_n + |k_2| P_{n-1} + \cdots\cdots + |k_n| P_1 \leq MQ_n$ and $\lim_{n \to \infty} \frac{k_n}{Q_n} = 0$.

(b) Show that for every k, the (c, k) matrix is a regular Nörlund mean.

6. (a) If f(n) is counting function of the second kind for a strongly regular matrix, then show that for any function of the form 0[f(n)] is a counting function of the first kind.

(b) Show that a regular (M) matrix is perfect.

7. (a) If $A = (a_{mn})$ and $B = (b_{mn})$ are regular matrices and A is μ_n-stronger than B, then show that A and B are ρ_n-consistent for some $\rho_n \to \infty$ $(\rho_n \to \mu_n)$.

(b) Define a truncated matrix and show that for a regular matrix $A = (a_{mn})$ there exists a sequence (ρ_n), $\rho_n \to \infty$ and a truncated matrix $B = (b_{mn})$ such that $\lim_{n \to \infty} \left(\sum_{n=1}^{\infty} a_{mn} s_n - \sum_{n=1}^{\infty} b_{mn} s_n \right) = 0$ for every (s_n), $s_n = O(\rho_n)$.

8. (a) Let $A = (a_{mn})$ and $B = (b_{mn})$ be regular matrices such that $A \supseteq B$. Then show that a unrformly bounded set of sequences E which