2015

Time : 4 hours

Full Marks : 100

The questions are of equal value.

*Answer any **five** questions from each Group.*

Symbols used have their usual meanings.

(LINEAR PROGRAMMING AND MEASURE AND INTEGRATION)

Group – A

(Full Marks : 50)

(LINEAR PROGRAMMING)

1. Define the following terms :

   (a) Basic feasible solution

   (b) Slack and surplus variables

   (c) Degenerate basic solution

   (d) Objective function

2. Prove that any convex combination of K different optimum solutions to an LPP is again an optimum solution to the problem.

(Turn over)

3. Obtain all the basic solutions to the following system of linear equations :

$$x_1 + 2x_2 + x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5$$

4. Using simplex method to solve the following LPP :

Max : $Z = x_1 + 2x_2$

Subject to : $-x_1 + 2x_2 \leq 8$

$x_1 + 2x_2 \leq 12$

$x_1 - 2x_2 \leq 3$

and $x_j \geq 0$, for $j = 1, 2$

5. Using Penalty on Big-m method to solve the following LPP :

Minimize : $Z = 2x_1 + x_2$

Subject to : $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

and $x_j \geq 0$, for $j = 1, 2$

6. Using revised simplex method to solve the following LPP :

Min : $Z = x_1 + x_2$

Subject to : $x_1 + 2x_2 \geq 7$

$4x_1 + x_2 \geq 6$

and $x_j \geq 0$, for $j = 1, 2$

7. Find the dual of the problem :

Max : $Z = x_1 - x_2 + 3x_3$

Subject to : $x_1 + x_2 + x_3 \leq 10$

$2x_1 - x_3 \leq 2$

$2x_1 - 2x_2 + 3x_3 \leq 6$

and $x_j \geq 0$, for $j = 1, 2, 3$

8. If for feasible solution X_o, W_o to the primal and dual respectively and $cX_o = W_o b$, then X_o is an optimal solution to the primal and W_o is optimal to the dual.

9. A necessary and sufficient condition for the existence of a feasible solution to a transportation problem is that $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, where a_i is the quantity of commodity available at origin i, and b_j

is the quantity of commodity needed at destination j.

10. Solve the transportation problem :

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

Group – B

(Full Marks : 50)

(MEASURE AND INTEGRATION)

11. Prove that the union of two measurable sets is measurable.

12. Let $E \subset (0, 1)$ be a measurable set. Then for each $y \in (0, 1)$ the set $E + y$ is measurable and $m(E + y) = mE$.

13. If f is a measurable function and $f = g$ a.e., then g is measurable.

14. Show that every Borel set is measurable.

15. Let $\langle f_n \rangle$ be an increasing sequence of non-

negative measurable functions, and let $f = \lim f_n$ a.e. Then prove that $\int f = \lim \int f_n$.

16. If f is integrable on [a, b], the function F defined by $F(x) = \int_a^x f(t)\, dt$ is a continuous functions of bounded variation on [a, b].

17. A function f is of bounded variation on [a, b] if and only if f is the difference of two monotone real valued functions on [a, b].

18. State and prove Holder Inequality.

19. Prove that ℓ^p is complete, for $1 \le p < \infty$.

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