7. Prove that for any two space X and Y, the homotopy relation $\simeq$ in an equivalence relation on the set C(X, Y) of continuous maps from X to Y.

8. Let K be an oriented complex and K its augmented complex. Then show that $H_p(\tilde{K}) \approx H_p(K)$ for all $p > 0$ and $H_0(K) \approx H_0(\tilde{K}) \otimes Z$.

9. Let C and D be chain complexes and ϕ, ψ : $C \to D$ chain homotopic chain maps. Then show that the induced homomorphisms ϕ_*, ψ_* : $H(C) \to H(D)$ are identical.

10. Let K and L be oriented geometric complexes and $\phi, \psi : |K| \to |L|$ simplical maps in the same contiguity class. Then show that the induced chain map $\phi^o, \psi^o : C(K) \to C(L)$ are chain homotopic. In particular, the induced chain maps ϕ_*, ψ_* : $H(K) \to H(L)$ is homology are identical.

2015

Time : 4 hours

Full Marks : 100

The questions are of equal value.

*Answer any **five** questions.*

*Answer any **five** questions from any **two** Groups as per specialisation.*

Group – A

(GRAPH THEORY)

Full Marks : 50

1. Define the following :

   (a) Isomorphism of graphs

   (b) Adjacency matrix

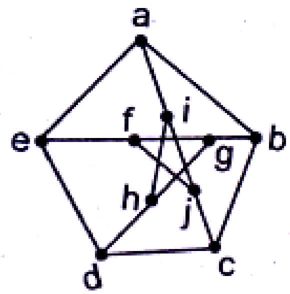
   (c) Bipartite graph

   (d) Planar graph

2. Prove that in a graph G, every u – v path contains a simple u – v path.

3. Prove that a simple non-directed graph G is a tree iff G is connected and contains no cycles.

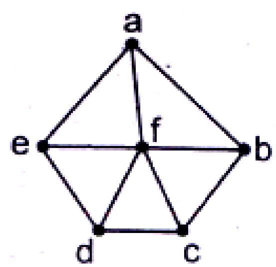
4. Prove that there is no Hamiltonian cycle in the following Petersen graph.



5. Prove that a complele bipartite graph $K_{m,n}$ is planar iff $m \leq 2$ or $n \leq 2$.

6. Show that a plane connected graph with less than 30 edges has a vertex of degree ≤ 4.

7. Find the chromatic number of the wheel graph.



8. Suppose that G is a planar graph with n vertices each of which has degree at most 5 and at least one vertex of degree 4. Use induction to prove that G is 4-colourable.

9. If F is a flow in a transport network (G, K) and if $(X, \bar{X})$ is any S – D cut, then prove that :
   (a) $|F| = F(X, \bar{X}) - F(\bar{X}, X)$ and consequently.
   (b) $|F| \leq K(X, \bar{X})$.

10. If K(e) = 1 for each edge 'e' in a transport network (G, K), find another description for the capacity of a cut $(X, \bar{X})$.

Group – B

(PROGRAMMING IN C)

Full Marks : 50

1. (a) Convert the equation $R = \dfrac{2v + 6{\cdot}22\,(c + d)}{g + v}$ into corresponding C statement.
   (b) If a five digit number is input through the key-board, write a program to calculate the sum of its digits.

2. Discuss about "if-else" and nested "if-else" statements with examples.

3. Explain "break" and "continue" statement with examples.

4. (a) Write a program to find the factorial value of any number.

   (b) Write a program to determine whether an integer is odd or even.

5. Write a recursive function to obtain the first 25 numbers of a Fibonacci sequence.

6. Discuss the pointer notation.

7. Describe different storage clases in 'C'.

8. What is Macro expansion ? Write down the marco definitions of :

   (a) To test whether a character entered is a small case letter or not.

   (b) To test whether a character entered is a upper case letter or not.

9. Write a program to sort all the elements of a 4 × 4 matrix.

10. Write a program to add two 6 × 6 matrices.

Group – C

(ALGEBRAIC TOPOLOGY)

Full Marks : 50

1. Show that every non-empty closed subset A of a Cantor space C is a retract of C. Prove that the diameter of a simplex $s_p = (a_0, a_1, \cdots, a_p)$ is the length of its longest edge.

2. State and prove Browser fixed theorem.

3. If $s_p = (a_0, a_1, \cdots\cdots, a_p)$ is a p-simplex in R^n and $\gamma \in R^n$, then prove that $(\gamma, |K(s_p)|)$ is in general position if the set $\{a_1 \cdots\cdots, a_p, \gamma\}$ is in geometrical independent.

4. Prove that a connected graph G is Euler if and only if the degree of every vertex is even.

5. Let f_0 and f_1 be homotopic maps of X into Y and g_0 and g_1 homotopic maps of Y and Z. Then show that $g_0 o f_0$ and $g_1 o f_1$ are homotopic maps of X into Z.

6. Show that any continuous map of S^m into S^n, where $m < n$ is null-homotopic.