15. Derive the formula for the first derivative of $y = f(x)$ of $O(h^2)$ using forward difference approximation.

16. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Gauss-Legendre three point formula. Compare with the exact solution.

17. Evaluate the integral $\int_0^1 \frac{dx}{1+x}$ using composite Simpson's rule, with 2, 4 and 8 equal subintervals.

18. Evaluate the integral $\int_1^2 \int_1^2 \frac{dx\,dy}{x+y}$ using the trapezoidal rule with $h = k = 0.5$ and $h = k = 0.25$. Improve the estimate using Romberg integration.

19. Solve the 1VP $u^1 = -2tu^2$, $u(0) = 1$ with $h = 0.2$, on the interval [0, 0.4], using the backward Euler method.

20. Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval (0, 0.4). Use the fourth order classical Runge-Kutta method. Compare with the exact solution.

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2015

Time : 4 hours

Full Marks : 100

The questions are of equal value.

*Answer any **five** questions from each Group.*

Symbols used have their usual meanings.

Group – A

Marks : 50

(GENERAL TOPOLOGY)

1. Let Y be a subspace of X. If A is closed in Y and Y is closed in X, then A is closed in X.

2. Show that if U is open in X, and A is closed in X, then U – A is open in X and A – U is closed in X.

3. Let $f : A \to X \times Y$ be given by the equation $f(a) = (f_1(a), f_2(a))$. Then f is continuous if and only if the function $f_1 : A \to X$ and $f_2 : A \to Y$ are contiuous.

4. Let A be a connected subset of X. If $A \subset B \subset \bar{A}$, then show that B in also connected.

5. Prove that every compact subset of a Housdorff space is closed.

6. Prove that a countable product of first countable spaces is first-countable.

7. Let X be a topological space. Let one point sets in X be closed. X is normal if and only if given a closed set A and an open set U containing A, there is an open set V containing A such that $\bar{V} \subset U$.

8. Prove that a product of regular spaces is regular.

9. Prove that if each space X_α in a Housdorff space, then ΠX_α is a Housdorff space in both the box and product topology.

10. Show that the topology is the smallest topology on ΠX_α relative to which each projection function π_β is continuous.

Group – B

Marks : 50

(NUMERICAL ANALYSIS)

11. Given that f(0) = 1, f(1) = 3, f(3) = 55, find the unique polynomial of degree 2 or less, which fits the given data. Find the bond on the error.

12. Using the following values of f(x) and f′(x) :

x	f(x)	f′(x)
– 1	1	– 5
0	1	1
1	3	7

Estimate the values of f(– 0.5) and f(0.5) using piecewise cubic Hermite interpolation.

13. Obtain the least squares polynomial approximation of degree two for $f(x) = x^3$ on (0, 1) with w(x) = 1.

14. Using the Gram-Schmidt Orthogonalization process, complete the first three orthogonal polynomials $P_0(x)$, $P_1(x)$, $P_2(x)$, which are orthogonal on (0, 1) with respect to the weright function w(x) = 1.