9. (a) Describe the nature of trajectories in the phase space for the following systems :

$x_2' = x, x_2' = x_2.$

(b) Let the matirx A in $x' = Ax$, $0 < t \leq \infty$ (1) where A is an $n \times n$ constant matrix and $x \in R^n$ be stable. Then for any solution $x(t)$ of (1),

$$\lim_{t \to \infty} \|x(t)\| = 0.$$

10. (a) Let $a(t)$ be non-decreasing continuous function such that $a(t) \to \infty$ as $t \to \infty$. Then all solutions of $x'' + a(t)x = 0$, $0 \leq t < \infty$ where a is a continuous function on $0 \leq t < \infty$ are bounded.

(b) The null solution of $x' = A(t)x$ is asymptotically stable if and only if $\|\phi(t)\| \to 0$ as $t \to \infty$.

2015

Time : 4 hours

Full Marks : 100

The questions are of equal value.

Answer any ***five*** *questions.*

Symbols used have their usual meanings.

(ORDINARY DIFFERENTIAL EQUATIONS)

1. (a) Solve the equation $x^2 t dx - (x^3 + t^3) dt = 0$.

(b) Test the equation $e^t dx + (xe^t + 2t)dt = 0$ for exactness and solve if it in exact.

2. (a) Solve the IVP :

$x''' + x'' = 0$; $x(0) = 1$, $x'(0) = 0$, $x''(0) = 1$.

(b) Solve $x'' + x = \tan t$.

3. (a) Solve : $x_1' = 2x_1 + x_2$

$x_2' = 3x_1 + 4x_2$

(b) Prove that the set of all solutions of the system $x' = A(t)x$ on I forms an n-dimensional vector space over the field of complex mumbers.

4. (a) Prove that the necessary and sufficient condition for the system $x' = Ax$ to admit a non-zero periodic solution of period w in that $E - e^{Aw}$ in singular, where E in the identity matrix.

(b) Find a Fundamental Matrix for the system

$$x' = Ax, \text{ where } A = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$$

5. (a) Show that the following function satisfy the Lipschitz condition in the rectangle indicated and find the Lipschitz constant :

$$f(t, x) = e^t \sin x, \ |x| \le 2\pi, \ |t| \le 1.$$

(b) Determine the constant L, K and h for the IVP

$x^1 = x^2$, $x(0) = 1$, $R = \{(t, x) : |t| \le 2, |x - 1| \le 2\}$

by using Picard's theorem.

6. (a) Let $I = [t_o, t_o + h]$, $v, w \in c'[I, R]$ be lower and upper solutions of $x' = f(t, x)$, $x(t_o) = x_o$ such that $v(t) \le w(t)$ on I and $f \in c\,[\Omega, R]$. Then, there

exists a solution x(t) of $x' = f(t, x)$, $x(t_o) = x_o$ such that $v(t) \le x(t) \le w(t)$ on I.

(b) Let $f \in c\,[I \times R, R]$, vo, wo, be lower and upper solutions of $x' = f(t, x)$, $x(t_o) = x_o$ such that $v_o \le w_o$ on $I = [t_o, t_o + h]$. Suppose that $f(t, x) - f(t, y) \ge -m(x - y)$ for $v_o \le y \le x \le w_o$ and $M \ge o$. Then there exits montone sequences $\{v_n\}$, $\{w_n\}$ such that $v_n \to v$ and $w_n \to w$ as $n \to \infty$ uniformly and monotonically on I and that v, w are minimal and maximal solutions of $x' = f(t, x)$, $x(t_o) = x_o$ respectively.

7. (a) Find the eigen values and eigen functions of the equation :

$$x'' + \lambda x = 0 \ ; \ 0 \le t \le \pi, \ x'(0) = x'(\pi) = 0$$

(b) Solve the BVP $x'' = t$, $x(0) = x(1) = 0$.

8. (a) State and prove Sturm's separation theorem for a equation $x'' + a(t)\,x' + b(t)\,x = 0$, $t \ge 0$ where a(t), b(t) are real-valued continuous fuctions on $(0, \infty)$.

(b) Prove that the Euler's equation $x'' + \dfrac{k}{t^2}\,x = 0$ in non-oscillatory if $k \le \dfrac{1}{4}$.